CLASS XII INTEGRALS CHAPTER 7

EX. 7.1 SOLUTIONS

Find an anti derivative (or integral) of the following functions by the method of inspection.

1. $\sin 2x$
2. $\cos 3x$
3. e^{2x}
4. $(ax + b)^2$
5. $\sin 2x - 4e^{3x}$

ANS :

1.

The anti derivative of $\sin 2x$ is a function of x whose derivative is $\sin 2x$.

It is known that,

$$\frac{d}{dx}(\cos 2x) = -2\sin 2x$$

$$\Rightarrow \sin 2x = -\frac{1}{2}\frac{d}{dx}(\cos 2x)$$

$$\therefore \sin 2x = \frac{d}{dx}\left(-\frac{1}{2}\cos 2x\right)$$

Therefore, the anti derivative of $\sin 2x$ is $-\frac{1}{2}\cos 2x$

2.

The anti derivative of $\cos 3x$ is a function of x whose derivative is $\cos 3x$.

It is known that,

$$\frac{d}{dx}(\sin 3x) = 3\cos 3x$$

$$\Rightarrow \cos 3x = \frac{1}{3}\frac{d}{dx}(\sin 3x)$$

$$\therefore \cos 3x = \frac{d}{dx}\left(\frac{1}{3}\sin 3x\right)$$

Therefore, the anti derivative of $\cos 3x$ is $\frac{1}{3}\sin 3x$.

3.

The anti derivative of e^{2x} is the function of x whose derivative is e^{2x}.

It is known that,

$$\frac{d}{dx}\left(e^{2x}\right)=2e^{2x}$$
$$\Rightarrow e^{2x}=\frac{1}{2}\frac{d}{dx}\left(e^{2x}\right)$$
$$\therefore e^{2x}=\frac{d}{dx}\left(\frac{1}{2}e^{2x}\right)$$

Therefore, the anti derivative of e^{2x} is $\frac{1}{2}e^{2x}$.

4.

The anti derivative of $(ax+b)^2$ is the function of x whose derivative is $(ax+b)^2$.

It is known that,

$$\frac{d}{dx}(ax+b)^3=3a(ax+b)^2$$
$$\Rightarrow (ax+b)^2=\frac{1}{3a}\frac{d}{dx}(ax+b)^3$$
$$\therefore (ax+b)^2=\frac{d}{dx}\left(\frac{1}{3a}(ax+b)^3\right)$$

Therefore, the anti derivative of $(ax+b)^2$ is $\frac{1}{3a}(ax+b)^3$.

5.

The anti derivative of $\left(\sin 2x-4e^{3x}\right)$ is the function of x whose derivative is $\left(\sin 2x-4e^{3x}\right)$.

It is known that,

$$\frac{d}{dx}\left(-\frac{1}{2}\cos 2x-\frac{4}{3}e^{3x}\right)=\sin 2x-4e^{3x}$$

Therefore, the anti derivative of $\left(\sin 2x-4e^{3x}\right)$ is $\left(-\frac{1}{2}\cos 2x-\frac{4}{3}e^{3x}\right)$.

6. $\int(4\,e^{3x}+1)\,dx$ 7. $\int x^2(1-\frac{1}{x^2})\,dx$ 8. $\int(ax^2+bx+c)\,dx$

ANS :

6.

$$\int\left(4e^{3x}+1\right)dx$$

$$=4\int e^{3x}dx+\int 1dx$$

$$=4\left(\frac{e^{3x}}{3}\right)+x+C$$

$$=\frac{4}{3}e^{3x}+x+C$$

7.

$$\int x^2\left(1-\frac{1}{x^2}\right)dx$$

$$=\int\left(x^2-1\right)dx$$

$$=\int x^2dx-\int 1dx$$

$$=\frac{x^3}{3}-x+C$$

8.

$$\int\left(ax^2+bx+c\right)dx$$

$$=a\int x^2dx+b\int xdx+c\int 1.dx$$

$$=a\left(\frac{x^3}{3}\right)+b\left(\frac{x^2}{2}\right)+cx+C$$

$$=\frac{ax^3}{3}+\frac{bx^2}{2}+cx+C$$

9. $\int (2x^2 + e^x)\, dx$ **10.** $\int \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 dx$ **11.** $\int \frac{x^3 + 5x^2 - 4}{x^2}\, dx$

ANS :

9.

$$\int \left(2x^2 + e^x\right) dx$$

$$= 2\int x^2 dx + \int e^x dx$$

$$= 2\left(\frac{x^3}{3}\right) + e^x + C$$

$$= \frac{2}{3}x^3 + e^x + C$$

10.

$$\int \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 dx$$

$$= \int \left(x + \frac{1}{x} - 2\right) dx$$

$$= \int x dx + \int \frac{1}{x} dx - 2\int 1.dx$$

$$= \frac{x^2}{2} + \log|x| - 2x + C$$

11.

$$\int \frac{x^3 + 5x^2 - 4}{x^2} dx$$

$$= \int \left(x + 5 - 4x^{-2}\right) dx$$

$$= \int x dx + 5\int 1.dx - 4\int x^{-2} dx$$

$$= \frac{x^2}{2} + 5x - 4\left(\frac{x^{-1}}{-1}\right) + C$$

$$= \frac{x^2}{2} + 5x + \frac{4}{x} + C$$

12. $\int \frac{x^3+3x+4}{\sqrt{x}}\,dx$ **13.** $\int \frac{x^3-x^2+x-1}{x-1}\,dx$ **14.** $\int (1-x)\sqrt{x}\,dx$

ANS :

12.

$$\int \frac{x^3+3x+4}{\sqrt{x}}dx$$

$$= \int\left(x^{\frac{5}{2}}+3x^{\frac{1}{2}}+4x^{-\frac{1}{2}}\right)dx$$

$$= \frac{x^{\frac{7}{2}}}{\frac{7}{2}}+\frac{3\left(x^{\frac{3}{2}}\right)}{\frac{3}{2}}+\frac{4\left(x^{\frac{1}{2}}\right)}{\frac{1}{2}}+C$$

$$= \frac{2}{7}x^{\frac{7}{2}}+2x^{\frac{3}{2}}+8x^{\frac{1}{2}}+C$$

$$= \frac{2}{7}x^{\frac{7}{2}}+2x^{\frac{3}{2}}+8\sqrt{x}+C$$

13.

On dividing, we obtain

$$= \int\left(x^2+1\right)dx$$

$$= \int x^2 dx + \int 1 dx$$

$$= \frac{x^3}{3}+x+C$$

14.

$$\int (1-x)\sqrt{x}dx$$

$$= \int\left(\sqrt{x}-x^{\frac{3}{2}}\right)dx$$

$$= \int x^{\frac{1}{2}}dx - \int x^{\frac{3}{2}}dx$$

$$= \frac{x^{\frac{3}{2}}}{\frac{3}{2}}-\frac{x^{\frac{5}{2}}}{\frac{5}{2}}+C$$

$$= \frac{2}{3}x^{\frac{3}{2}}-\frac{2}{5}x^{\frac{5}{2}}+C$$

15. $\int \sqrt{x}(3x^2 + 2x + 3)\, dx$

16. $\int (2x - 3\cos x + e^x)\, dx$

ANS :

15.

$$\int \sqrt{x}\left(3x^2 + 2x + 3\right)dx$$

$$= \int\left(3x^{\frac{5}{2}} + 2x^{\frac{3}{2}} + 3x^{\frac{1}{2}}\right)dx$$

$$= 3\int x^{\frac{5}{2}}dx + 2\int x^{\frac{3}{2}}dx + 3\int x^{\frac{1}{2}}dx$$

$$= 3\left(\frac{x^{\frac{7}{2}}}{\frac{7}{2}}\right) + 2\left(\frac{x^{\frac{5}{2}}}{\frac{5}{2}}\right) + 3\frac{\left(x^{\frac{3}{2}}\right)}{\frac{3}{2}} + C$$

$$= \frac{6}{7}x^{\frac{7}{2}} + \frac{4}{5}x^{\frac{5}{2}} + 2x^{\frac{3}{2}} + C$$

16.

$$\int\left(2x - 3\cos x + e^x\right)dx$$

$$= 2\int x dx - 3\int \cos x dx + \int e^x dx$$

$$= \frac{2x^2}{2} - 3\left(\sin x\right) + e^x + C$$

$$= x^2 - 3\sin x + e^x + C$$

17. $\int (2x^2 - 3\sin x + 5\sqrt{x})\, dx$

18. $\int \sec x\, (\sec x + \tan x)\, dx$

19. $\int \frac{\sec^2 x}{\operatorname{cosec}^2 x}\, dx$

20. $\int \frac{2 - 3\sin x}{\cos^2 x}\, dx.$

ANS :

17.

$$\int \left(2x^2 - 3\sin x + 5\sqrt{x}\right) dx$$

$$= 2\int x^2 dx - 3\int \sin x dx + 5\int x^{\frac{1}{2}} dx$$

$$= \frac{2x^3}{3} - 3(-\cos x) + 5\left(\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right) + C$$

$$= \frac{2}{3}x^3 + 3\cos x + \frac{10}{3}x^{\frac{3}{2}} + C$$

18.

$$\int \sec x(\sec x + \tan x)\, dx$$

$$= \int \left(\sec^2 x + \sec x \tan x\right) dx$$

$$= \int \sec^2 x dx + \int \sec x \tan x dx$$

$$= \tan x + \sec x + C$$

19.

$$\int \frac{\sec^2 x}{\cos ec^2 x} dx$$

$$= \int \frac{\frac{1}{\cos^2 x}}{\frac{1}{\sin^2 x}} dx$$

$$= \int \frac{\sin^2 x}{\cos^2 x} dx$$

$$= \int \tan^2 x dx$$

$$= \int \left(\sec^2 x - 1\right) dx$$

$$= \int \sec^2 x dx - \int 1 dx$$

$$= \tan x - x + C$$

20.

$$= \int \left(\frac{2}{\cos^2 x} - \frac{3\sin x}{\cos^2 x}\right) dx$$

$$= \int 2\sec^2 x dx - 3\int \tan x \sec x dx$$

$$= 2\tan x - 3\sec x + C$$

21. The anti derivative of $\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)$ equals

(A) $\frac{1}{3}x^{\frac{1}{3}}+2x^{\frac{1}{2}}+C$

(B) $\frac{2}{3}x^{\frac{2}{3}}+\frac{1}{2}x^{2}+C$

(C) $\frac{2}{3}x^{\frac{3}{2}}+2x^{\frac{1}{2}}+C$

(D) $\frac{3}{2}x^{\frac{3}{2}}+\frac{1}{2}x^{\frac{1}{2}}+C$

ANS :

$$\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)dx$$

$$=\int x^{\frac{1}{2}}dx+\int x^{-\frac{1}{2}}dx$$

$$=\frac{x^{\frac{3}{2}}}{\frac{3}{2}}+\frac{x^{\frac{1}{2}}}{\frac{1}{2}}+C$$

$$=\frac{2}{3}x^{\frac{3}{2}}+2x^{\frac{1}{2}}+C$$

Hence, the correct answer is C.

22. If $\frac{d}{dx}f(x) = 4x^3 - \frac{3}{x^4}$ such that $f(2) = 0$. Then $f(x)$ is

(A) $x^4 + \frac{1}{x^3} - \frac{129}{8}$

(B) $x^3 + \frac{1}{x^4} + \frac{129}{8}$

(C) $x^4 + \frac{1}{x^3} + \frac{129}{8}$

(D) $x^3 + \frac{1}{x^4} - \frac{129}{8}$

ANS :

It is given that,

$$\frac{d}{dx}f(x) = 4x^3 - \frac{3}{x^4}$$

$\therefore$Anti derivative of $4x^3 - \frac{3}{x^4} = f(x)$

$$\therefore f(x) = \int 4x^3 - \frac{3}{x^4}\,dx$$

$$f(x) = 4\int x^3 dx - 3\int (x^{-4})dx$$

$$\therefore f(x) = 4\left(\frac{x^4}{4}\right) - 3\left(\frac{x^{-3}}{-3}\right) + C$$

$$f(x) = x^4 + \frac{1}{x^3} + C$$

Also,

$$f(2) = 0$$

$$\therefore f(2) = (2)^4 + \frac{1}{(2)^3} + C = 0$$

$$\Rightarrow 16 + \frac{1}{8} + C = 0$$

$$\Rightarrow C = -\left(16 + \frac{1}{8}\right)$$

$$\Rightarrow C = \frac{-129}{8}$$

$$\therefore f(x) = x^4 + \frac{1}{x^3} - \frac{129}{8}$$

Hence, the correct answer is A.