

CLASS XII

CHAPTER 5 CONTINUITY AND DIFFERENTIABILITY

NCERT EX 5.6 SOLUTIONS

Question 1:

If x and y are connected parametrically by the equation, without eliminating the parameter, find $\frac{dy}{dx}$.

$$x = 2at^2, y = at^4$$

ANS:

The given equations are $x = 2at^2$ and $y = at^4$

$$\text{Then, } \frac{dx}{dt} = \frac{d}{dt}(2at^2) = 2a \cdot \frac{d}{dt}(t^2) = 2a \cdot 2t = 4at$$

$$\frac{dy}{dt} = \frac{d}{dt}(at^4) = a \cdot \frac{d}{dt}(t^4) = a \cdot 4 \cdot t^3 = 4at^3$$

$$\therefore \frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{4at^3}{4at} = t^2$$

Question 2:

If x and y are connected parametrically by the equation, without eliminating the parameter, find $\frac{dy}{dx}$.

$$x = a \cos \theta, y = b \cos \theta$$

ANS:

The given equations are $x = a \cos \theta$ and $y = b \cos \theta$

$$\text{Then, } \frac{dx}{d\theta} = \frac{d}{d\theta}(a \cos \theta) = a(-\sin \theta) = -a \sin \theta$$

$$\frac{dy}{d\theta} = \frac{d}{d\theta}(b \cos \theta) = b(-\sin \theta) = -b \sin \theta$$

$$\therefore \frac{dy}{dx} = \frac{\left(\frac{dy}{d\theta}\right)}{\left(\frac{dx}{d\theta}\right)} = \frac{-b \sin \theta}{-a \sin \theta} = \frac{b}{a}$$

Question 3:

If x and y are connected parametrically by the equation, without eliminating the parameter, find $\frac{dy}{dx}$.

$$x = \sin t, y = \cos 2t$$

ANS:

The given equations are $x = \sin t$ and $y = \cos 2t$

$$\text{Then, } \frac{dx}{dt} = \frac{d}{dt}(\sin t) = \cos t$$

$$\frac{dy}{dt} = \frac{d}{dt}(\cos 2t) = -\sin 2t \cdot \frac{d}{dt}(2t) = -2 \sin 2t$$

$$\therefore \frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{-2 \sin 2t}{\cos t} = \frac{-2 \cdot 2 \sin t \cos t}{\cos t} = -4 \sin t$$

Question 4:

If x and y are connected parametrically by the equation, without eliminating the parameter, find $\frac{dy}{dx}$.

$$x = 4t, y = \frac{4}{t}$$

ANS:

The given equations are $x = 4t$ and $y = \frac{4}{t}$

$$\frac{dx}{dt} = \frac{d}{dt}(4t) = 4$$

$$\frac{dy}{dt} = \frac{d}{dt}\left(\frac{4}{t}\right) = 4 \cdot \frac{d}{dt}\left(\frac{1}{t}\right) = 4 \cdot \left(\frac{-1}{t^2}\right) = \frac{-4}{t^2}$$

$$\therefore \frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{\left(\frac{-4}{t^2}\right)}{4} = \frac{-1}{t^2}$$

Question 5:

If x and y are connected parametrically by the equation, without eliminating the parameter, find $\frac{dy}{dx}$.

$$x = \cos \theta - \cos 2\theta, \quad y = \sin \theta - \sin 2\theta$$

ANS:

The given equations are $x = \cos \theta - \cos 2\theta$ and $y = \sin \theta - \sin 2\theta$

$$\begin{aligned}\text{Then, } \frac{dx}{d\theta} &= \frac{d}{d\theta}(\cos \theta - \cos 2\theta) = \frac{d}{d\theta}(\cos \theta) - \frac{d}{d\theta}(\cos 2\theta) \\ &= -\sin \theta - (-2 \sin 2\theta) = 2 \sin 2\theta - \sin \theta\end{aligned}$$

$$\begin{aligned}\frac{dy}{d\theta} &= \frac{d}{d\theta}(\sin \theta - \sin 2\theta) = \frac{d}{d\theta}(\sin \theta) - \frac{d}{d\theta}(\sin 2\theta) \\ &= \cos \theta - 2 \cos 2\theta\end{aligned}$$

$$\therefore \frac{dy}{dx} = \frac{\left(\frac{dy}{d\theta}\right)}{\left(\frac{dx}{d\theta}\right)} = \frac{\cos \theta - 2 \cos 2\theta}{2 \sin 2\theta - \sin \theta}$$

Question 6:

If x and y are connected parametrically by the equation, without eliminating the parameter, find $\frac{dy}{dx}$.

$$x = a(\theta - \sin \theta), \quad y = a(1 + \cos \theta)$$

ANS:

The given equations are $x = a(\theta - \sin \theta)$ and $y = a(1 + \cos \theta)$

$$\text{Then, } \frac{dx}{d\theta} = a \left[\frac{d}{d\theta}(\theta) - \frac{d}{d\theta}(\sin \theta) \right] = a(1 - \cos \theta)$$

$$\frac{dy}{d\theta} = a \left[\frac{d}{d\theta}(1) + \frac{d}{d\theta}(\cos \theta) \right] = a[0 + (-\sin \theta)] = -a \sin \theta$$

$$\therefore \frac{dy}{dx} = \frac{\left(\frac{dy}{d\theta}\right)}{\left(\frac{dx}{d\theta}\right)} = \frac{-a \sin \theta}{a(1 - \cos \theta)} = \frac{-2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}}{2 \sin^2 \frac{\theta}{2}} = \frac{-\cos \frac{\theta}{2}}{\sin \frac{\theta}{2}} = -\cot \frac{\theta}{2}$$

Question 7:

If x and y are connected parametrically by the equation, without eliminating the parameter, find $\frac{dy}{dx}$.

$$x = \frac{\sin^3 t}{\sqrt{\cos 2t}}, y = \frac{\cos^3 t}{\sqrt{\cos 2t}}$$

ANS:

The given equations are $x = \frac{\sin^3 t}{\sqrt{\cos 2t}}$ and $y = \frac{\cos^3 t}{\sqrt{\cos 2t}}$

$$\begin{aligned} \text{Then, } \frac{dx}{dt} &= \frac{d}{dt} \left[\frac{\sin^3 t}{\sqrt{\cos 2t}} \right] \\ &= \frac{\sqrt{\cos 2t} \cdot \frac{d}{dt}(\sin^3 t) - \sin^3 t \cdot \frac{d}{dt} \sqrt{\cos 2t}}{\cos 2t} \\ &= \frac{\sqrt{\cos 2t} \cdot 3 \sin^2 t \cdot \frac{d}{dt}(\sin t) - \sin^3 t \times \frac{1}{2\sqrt{\cos 2t}} \cdot \frac{d}{dt}(\cos 2t)}{\cos 2t} \\ &= \frac{3\sqrt{\cos 2t} \cdot \sin^2 t \cos t - \frac{\sin^3 t}{2\sqrt{\cos 2t}} \cdot (-2 \sin 2t)}{\cos 2t} \\ &= \frac{3 \cos 2t \sin^2 t \cos t + \sin^3 t \sin 2t}{\cos 2t \sqrt{\cos 2t}} \end{aligned}$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dt} \left[\frac{\cos^3 t}{\sqrt{\cos 2t}} \right] \\ &= \frac{\sqrt{\cos 2t} \cdot \frac{d}{dt}(\cos^3 t) - \cos^3 t \cdot \frac{d}{dt}(\sqrt{\cos 2t})}{\cos 2t} \\ &= \frac{\sqrt{\cos 2t} \cdot 3 \cos^2 t \cdot \frac{d}{dt}(\cos t) - \cos^3 t \cdot \frac{1}{2\sqrt{\cos 2t}} \cdot \frac{d}{dt}(\cos 2t)}{\cos 2t} \\ &= \frac{3\sqrt{\cos 2t} \cdot \cos^2 t (-\sin t) - \cos^3 t \cdot \frac{1}{2\sqrt{\cos 2t}} \cdot (-2 \sin 2t)}{\cos 2t} \end{aligned}$$

$$= \frac{-3 \cos 2t \cdot \cos^2 t \cdot \sin t + \cos^3 t \sin 2t}{\cos 2t \cdot \sqrt{\cos 2t}}$$

$$\begin{aligned} \therefore \frac{dy}{dx} &= \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{-3 \cos 2t \cdot \cos^2 t \cdot \sin t + \cos^3 t \sin 2t}{3 \cos 2t \sin^2 t \cos t + \sin^3 t \sin 2t} \\ &= \frac{-3 \cos 2t \cdot \cos^2 t \cdot \sin t + \cos^3 t (2 \sin t \cos t)}{3 \cos 2t \sin^2 t \cos t + \sin^3 t (2 \sin t \cos t)} \\ &= \frac{\sin t \cos t [-3 \cos 2t \cdot \cos t + 2 \cos^3 t]}{\sin t \cos t [3 \cos 2t \sin t + 2 \sin^3 t]} \\ &= \frac{[-3(2 \cos^2 t - 1) \cos t + 2 \cos^3 t]}{[3(1 - 2 \sin^2 t) \sin t + 2 \sin^3 t]} && \left[\begin{array}{l} \cos 2t = (2 \cos^2 t - 1), \\ \cos 2t = (1 - 2 \sin^2 t) \end{array} \right] \\ &= \frac{-4 \cos^3 t + 3 \cos t}{3 \sin t - 4 \sin^3 t} \\ &= \frac{-\cos 3t}{\sin 3t} && \left[\begin{array}{l} \cos 3t = 4 \cos^3 t - 3 \cos t, \\ \sin 3t = 3 \sin t - 4 \sin^3 t \end{array} \right] \\ &= -\cot 3t \end{aligned}$$

Question 8:

If x and y are connected parametrically by the equation, without eliminating the parameter, find $\frac{dy}{dx}$.

$$x = a \left(\cos t + \log \tan \frac{t}{2} \right), \quad y = a \sin t$$

ANS:

The given equations are $x = a \left(\cos t + \log \tan \frac{t}{2} \right)$ and $y = a \sin t$

$$\begin{aligned}\text{Then, } \frac{dx}{dt} &= a \cdot \left[\frac{d}{dt}(\cos t) + \frac{d}{dt} \left(\log \tan \frac{t}{2} \right) \right] \\ &= a \left[-\sin t + \frac{1}{\tan \frac{t}{2}} \cdot \frac{d}{dt} \left(\tan \frac{t}{2} \right) \right] \\ &= a \left[-\sin t + \cot \frac{t}{2} \cdot \sec^2 \frac{t}{2} \cdot \frac{d}{dt} \left(\frac{t}{2} \right) \right] \\ &= a \left[-\sin t + \frac{\cos \frac{t}{2}}{\sin \frac{t}{2}} \times \frac{1}{\cos^2 \frac{t}{2}} \times \frac{1}{2} \right] \\ &= a \left[-\sin t + \frac{1}{2 \sin \frac{t}{2} \cos \frac{t}{2}} \right] \\ &= a \left(-\sin t + \frac{1}{\sin t} \right) \\ &= a \left(\frac{-\sin^2 t + 1}{\sin t} \right) \\ &= a \frac{\cos^2 t}{\sin t}\end{aligned}$$

$$\frac{dy}{dt} = a \frac{d}{dt}(\sin t) = a \cos t$$

$$\therefore \frac{dy}{dx} = \frac{\left(\frac{dy}{dt} \right)}{\left(\frac{dx}{dt} \right)} = \frac{a \cos t}{\left(a \frac{\cos^2 t}{\sin t} \right)} = \frac{\sin t}{\cos t} = \tan t$$

Question 9:

If x and y are connected parametrically by the equation, without eliminating the parameter, find $\frac{dy}{dx}$.

$$x = a \sec \theta, \quad y = b \tan \theta$$

ANS:

The given equations are $x = a \sec \theta$ and $y = b \tan \theta$

$$\text{Then, } \frac{dx}{d\theta} = a \cdot \frac{d}{d\theta}(\sec \theta) = a \sec \theta \tan \theta$$

$$\frac{dy}{d\theta} = b \cdot \frac{d}{d\theta}(\tan \theta) = b \sec^2 \theta$$

$$\therefore \frac{dy}{dx} = \frac{\left(\frac{dy}{d\theta}\right)}{\left(\frac{dx}{d\theta}\right)} = \frac{b \sec^2 \theta}{a \sec \theta \tan \theta} = \frac{b}{a} \sec \theta \cot \theta = \frac{b \cos \theta}{a \cos \theta \sin \theta} = \frac{b}{a} \times \frac{1}{\sin \theta} = \frac{b}{a} \operatorname{cosec} \theta$$

Question 10:

If x and y are connected parametrically by the equation, without eliminating the parameter, find $\frac{dy}{dx}$.

$$x = a(\cos \theta + \theta \sin \theta), \quad y = a(\sin \theta - \theta \cos \theta)$$

ANS:

The given equations are $x = a(\cos \theta + \theta \sin \theta)$ and $y = a(\sin \theta - \theta \cos \theta)$

$$\begin{aligned} \text{Then, } \frac{dx}{d\theta} &= a \left[\frac{d}{d\theta} \cos \theta + \frac{d}{d\theta}(\theta \sin \theta) \right] = a \left[-\sin \theta + \theta \frac{d}{d\theta}(\sin \theta) + \sin \theta \frac{d}{d\theta}(\theta) \right] \\ &= a[-\sin \theta + \theta \cos \theta + \sin \theta] = a\theta \cos \theta \end{aligned}$$

$$\begin{aligned} \frac{dy}{d\theta} &= a \left[\frac{d}{d\theta}(\sin \theta) - \frac{d}{d\theta}(\theta \cos \theta) \right] = a \left[\cos \theta - \left\{ \theta \frac{d}{d\theta}(\cos \theta) + \cos \theta \cdot \frac{d}{d\theta}(\theta) \right\} \right] \\ &= a[\cos \theta + \theta \sin \theta - \cos \theta] \\ &= a\theta \sin \theta \end{aligned}$$

$$\therefore \frac{dy}{dx} = \frac{\left(\frac{dy}{d\theta}\right)}{\left(\frac{dx}{d\theta}\right)} = \frac{a\theta \sin \theta}{a\theta \cos \theta} = \tan \theta$$

Question 11:

If $x = \sqrt{a^{\sin^{-1} t}}$, $y = \sqrt{a^{\cos^{-1} t}}$, show that $\frac{dy}{dx} = -\frac{y}{x}$

ANS:

The given equations are $x = \sqrt{a^{\sin^{-1} t}}$ and $y = \sqrt{a^{\cos^{-1} t}}$

$$x = \sqrt{a^{\sin^{-1} t}} \text{ and } y = \sqrt{a^{\cos^{-1} t}}$$

$$\Rightarrow x = \left(a^{\sin^{-1} t}\right)^{\frac{1}{2}} \text{ and } y = \left(a^{\cos^{-1} t}\right)^{\frac{1}{2}}$$

$$\Rightarrow x = a^{\frac{1}{2}\sin^{-1} t} \text{ and } y = a^{\frac{1}{2}\cos^{-1} t}$$

Consider $x = a^{\frac{1}{2}\sin^{-1} t}$

Taking logarithm on both the sides, we obtain

$$\log x = \frac{1}{2} \sin^{-1} t \log a$$

$$\therefore \frac{1}{x} \cdot \frac{dx}{dt} = \frac{1}{2} \log a \cdot \frac{d}{dt}(\sin^{-1} t)$$

$$\Rightarrow \frac{dx}{dt} = \frac{x}{2} \log a \cdot \frac{1}{\sqrt{1-t^2}}$$

$$\Rightarrow \frac{dx}{dt} = \frac{x \log a}{2\sqrt{1-t^2}}$$

Then, consider $y = a^{\frac{1}{2}\cos^{-1} t}$

Taking logarithm on both the sides, we obtain

$$\log y = \frac{1}{2} \cos^{-1} t \log a$$

$$\therefore \frac{1}{y} \cdot \frac{dy}{dt} = \frac{1}{2} \log a \cdot \frac{d}{dt}(\cos^{-1} t)$$

$$\Rightarrow \frac{dy}{dt} = \frac{y \log a}{2} \cdot \left(\frac{-1}{\sqrt{1-t^2}}\right)$$

$$\Rightarrow \frac{dy}{dt} = \frac{-y \log a}{2\sqrt{1-t^2}}$$

$$\therefore \frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{\left(\frac{-y \log a}{2\sqrt{1-t^2}}\right)}{\left(\frac{x \log a}{2\sqrt{1-t^2}}\right)} = -\frac{y}{x}$$

Hence, proved.