CLASS XII

CHAPTER 5 CONTINUITY AND DIFFERNENTIABILITY

NCERT EX 5.2 SOLUTIONS

Question 1:

Differentiate the functions with respect to x.

$\sin\left(x^2+5\right)$

ANS:

Let $f(x)=\sin\left(x^2+5\right)$, $u(x)=x^2+5$, and $v(t)=\sin t$

Then, $(vou)(x)=v(u(x))=v\left(x^2+5\right)=\tan\left(x^2+5\right)=f(x)$

Thus, f is a composite of two functions.

Put $t=u(x)=x^2+5$

Then, we obtain

$$\frac{dv}{dt}=\frac{d}{dt}(\sin t)=\cos t=\cos\left(x^2+5\right)$$

$$\frac{dt}{dx}=\frac{d}{dx}\left(x^2+5\right)=\frac{d}{dx}\left(x^2\right)+\frac{d}{dx}(5)=2x+0=2x$$

Therefore, by chain rule, $\frac{df}{dx}=\frac{dv}{dt}\cdot\frac{dt}{dx}=\cos\left(x^2+5\right)\times 2x=2x\cos\left(x^2+5\right)$

Alternate method

$$\begin{aligned}\frac{d}{dx}\left[\sin\left(x^2+5\right)\right]&=\cos\left(x^2+5\right)\cdot\frac{d}{dx}\left(x^2+5\right)\\&=\cos\left(x^2+5\right)\cdot\left[\frac{d}{dx}\left(x^2\right)+\frac{d}{dx}(5)\right]\\&=\cos\left(x^2+5\right)\cdot[2x+0]\\&=2x\cos\left(x^2+5\right)\end{aligned}$$

Question 2:

Differentiate the functions with respect to x.

$\cos(\sin x)$

ANS:

Let $f(x)=\cos(\sin x), u(x)=\sin x,$ and $v(t)=\cos t$

Then, $(vou)(x)=v(u(x))=v(\sin x)=\cos(\sin x)=f(x)$

Thus, f is a composite function of two functions.

Put $t=u(x)=\sin x$

$$\therefore \frac{dv}{dt}=\frac{d}{dt}[\cos t]=-\sin t=-\sin(\sin x)$$

$$\frac{dt}{dx}=\frac{d}{dx}(\sin x)=\cos x$$

By chain rule, $\frac{df}{dx}=\frac{dv}{dt}\cdot\frac{dt}{dx}=-\sin(\sin x)\cdot\cos x=-\cos x\sin(\sin x)$

Alternate method

$$\frac{d}{dx}[\cos(\sin x)]=-\sin(\sin x)\cdot\frac{d}{dx}(\sin x)=-\sin(\sin x)\cdot\cos x=-\cos x\sin(\sin x)$$

Question 3:

Differentiate the functions with respect to x.

$\sin(ax+b)$

ANS:

$$\begin{aligned}\frac{d}{dx}[\sin(ax+b)]&=\cos(ax+b).\frac{d}{dx}(ax+b)\\&=\cos(ax+b)\cdot\left[\frac{d}{dx}(ax)+\frac{d}{dx}(b)\right]\\&=\cos(ax+b).(a+0)\\&=a\cos(ax+b)\end{aligned}$$

Question 4:

Differentiate the functions with respect to x.

$\sec\left(\tan\left(\sqrt{x}\right)\right)$

ANS:

$$\frac{d}{dx}\left[\sec\left(\tan\sqrt{x}\right)\right]=\sec\left(\tan\sqrt{x}\right).\tan\left(\tan\sqrt{x}\right)\cdot\frac{d}{dx}\left(\tan\sqrt{x}\right)$$
$$=\sec\left(\tan\sqrt{x}\right)\cdot\tan\left(\tan\sqrt{x}\right)\cdot\sec^2\left(\sqrt{x}\right)\cdot\frac{d}{dx}\left(\sqrt{x}\right)$$
$$=\sec\left(\tan\sqrt{x}\right)\cdot\tan\left(\tan\sqrt{x}\right)\cdot\sec^2\left(\sqrt{x}\right)\cdot\frac{1}{2\sqrt{x}}$$
$$=\frac{\sec\left(\tan\sqrt{x}\right)\cdot\tan\left(\tan\sqrt{x}\right)\sec^2\left(\sqrt{x}\right)}{2\sqrt{x}}$$

Question 5:

Differentiate the functions with respect to x.

$$\frac{\sin(ax+b)}{\cos(cx+d)}$$

ANS:

The given function is $f(x)=\frac{\sin(ax+b)}{\cos(cx+d)}=\frac{g(x)}{h(x)}$, where $g(x) = \sin(ax + b)$ and

$h(x) = \cos(cx + d)$

$$\therefore f'=\frac{g'h-gh'}{h^2}$$

Consider $g(x)=\sin(ax+b)$

Let $u(x)=ax+b, v(t)=\sin t$

Then, $(vou)(x)=v(u(x))=v(ax+b)=\sin(ax+b)=g(x)$

$\therefore g$ is a composite function of two functions, u and v.

Put $t=u(x)=ax+b$

$$\frac{dv}{dt}=\frac{d}{dt}(\sin t)=\cos t=\cos(ax+b)$$
$$\frac{dt}{dx}=\frac{d}{dx}(ax+b)=\frac{d}{dx}(ax)+\frac{d}{dx}(b)=a+0=a$$

Therefore, by chain rule, we obtain

$$g' = \frac{dg}{dx} = \frac{dv}{dt} \cdot \frac{dt}{dx} = \cos(ax+b) \cdot a = a\cos(ax+b)$$

Consider $h(x) = \cos(cx+d)$

Let $p(x) = cx+d$, $q(y) = \cos y$

Then, $(qop)(x) = q(p(x)) = q(cx+d) = \cos(cx+d) = h(x)$

$\therefore h$ is a composite function of two functions, p and q.

Put $y = p(x) = cx + d$

$$\frac{dq}{dy} = \frac{d}{dy}(\cos y) = -\sin y = -\sin(cx+d)$$

$$\frac{dy}{dx} = \frac{d}{dx}(cx+d) = \frac{d}{dx}(cx) + \frac{d}{dx}(d) = c$$

Therefore, by chain rule, we obtain

$$h' = \frac{dh}{dx} = \frac{dq}{dy} \cdot \frac{dy}{dx} = -\sin(cx+d) \times c = -c\sin(cx+d)$$

$$\therefore f' = \frac{a\cos(ax+b) \cdot \cos(cx+d) - \sin(ax+b)\{-c\sin(cx+d)\}}{[\cos(cx+d)]^2}$$

$$= \frac{a\cos(ax+b)}{\cos(cx+d)} + c\sin(ax+b) \cdot \frac{\sin(cx+d)}{\cos(cx+d)} \times \frac{1}{\cos(cx+d)}$$

$$= a\cos(ax+b)\sec(cx+d) + c\sin(ax+b)\tan(cx+d)\sec(cx+d)$$

Question 6:

Differentiate the functions with respect to x.

$\cos x^3 . \sin^2(x^5)$

ANS:

$$\frac{d}{dx}\left[\cos x^3 \cdot \sin^2(x^5)\right] = \sin^2(x^5) \times \frac{d}{dx}(\cos x^3) + \cos x^3 \times \frac{d}{dx}\left[\sin^2(x^5)\right]$$

$$= \sin^2(x^5) \times (-\sin x^3) \times \frac{d}{dx}(x^3) + \cos x^3 \times 2\sin(x^5) \cdot \frac{d}{dx}\left[\sin x^5\right]$$

$$= -\sin x^3 \sin^2(x^5) \times 3x^2 + 2\sin x^5 \cos x^3 \cdot \cos x^5 \times \frac{d}{dx}(x^5)$$

$$= -3x^2 \sin x^3 \cdot \sin^2(x^5) + 2\sin x^5 \cos x^5 \cos x^3 \cdot \times 5x^4$$

$$= 10x^4 \sin x^5 \cos x^5 \cos x^3 - 3x^2 \sin x^3 \sin^2(x^5)$$

Question 7:

Differentiate the functions with respect to x.

$$2\sqrt{\cot\left(x^2\right)}$$

ANS:

$$\frac{d}{dx}\left[2\sqrt{\cot\left(x^2\right)}\right]$$

$$= 2\cdot\frac{1}{2\sqrt{\cot\left(x^2\right)}}\times\frac{d}{dx}\left[\cot\left(x^2\right)\right]$$

$$= \sqrt{\frac{\sin\left(x^2\right)}{\cos\left(x^2\right)}}\times -\operatorname{cosec}^2\left(x^2\right)\times\frac{d}{dx}\left(x^2\right)$$

$$= -\sqrt{\frac{\sin\left(x^2\right)}{\cos\left(x^2\right)}}\times\frac{1}{\sin^2\left(x^2\right)}\times(2x)$$

$$= \frac{-2x}{\sqrt{\cos x^2}\sqrt{\sin x^2}\,\sin x^2}$$

$$= \frac{-2\sqrt{2}\,x}{\sqrt{2\sin x^2\cos x^2}\,\sin x^2}$$

$$= \frac{-2\sqrt{2}\,x}{\sin x^2\sqrt{\sin 2x^2}}$$

Question 8:

Differentiate the functions with respect to x.

$$\cos\left(\sqrt{x}\right)$$

ANS:

$$\frac{d}{dx}\left[\cos\left(\sqrt{x}\right)\right] = -\sin\left(\sqrt{x}\right)\cdot\frac{d}{dx}\left(\sqrt{x}\right)$$

$$= -\sin\left(\sqrt{x}\right)\times\frac{d}{dx}\left(x^{\frac{1}{2}}\right)$$

$$= -\sin\sqrt{x}\times\frac{1}{2}x^{-\frac{1}{2}}$$

$$= \frac{-\sin\sqrt{x}}{2\sqrt{x}}$$

Question 9:

Prove that the function f given by

$f(x)=|x-1|,\ x\in\mathbf{R}$ is notdifferentiable at $x=1$.

ANS:

The given function is $f(x)=|x-1|,\ x\in\mathbf{R}$

It is known that a function f is differentiable at a point $x=c$ in its domain if both

$\lim\limits_{h\to 0^-}\frac{f(c+h)-f(c)}{h}$ and $\lim\limits_{h\to 0^+}\frac{f(c+h)-f(c)}{h}$ are finite and equal.

To check the differentiability of the given function at $x=1$,

consider the left hand limit of f at $x=1$

$$\lim_{h\to 0^-}\frac{f(1+h)-f(1)}{h}=\lim_{h\to 0^-}\frac{|1+h-1|-|1-1|}{h}$$

$$=\lim_{h\to 0^-}\frac{|h|-0}{h}=\lim_{h\to 0^-}\frac{-h}{h} \qquad (h<0\Rightarrow|h|=-h)$$

$$=-1$$

Consider the right hand limit of f at $x=1$

$$\lim_{h\to 0^+}\frac{f(1+h)-f(1)}{h}=\lim_{h\to 0^+}\frac{|1+h-1|-|1-1|}{h}$$

$$=\lim_{h\to 0^+}\frac{|h|-0}{h}=\lim_{h\to 0^+}\frac{h}{h} \qquad (h>0\Rightarrow|h|=h)$$

$$=1$$

Since the left and right hand limits of f at $x=1$ are not equal, f is not differentiable at $x=1$

Question 10:

Prove that the greatest integer function defined by $f(x)=[x], 0<x<3$ is not

differentiable at $x=1$ and $x=2$.

ANS:

The given function f is $f(x) = [x], 0 < x < 3$

It is known that a function f is differentiable at a point $x = c$ in its domain if both

$\lim_{h \to 0^-} \frac{f(c+h) - f(c)}{h}$ and $\lim_{h \to 0^+} \frac{f(c+h) - f(c)}{h}$ are finite and equal.

To check the differentiability of the given function at $x = 1$, consider the left hand limit of f at $x = 1$

$$\lim_{h \to 0^-} \frac{f(1+h) - f(1)}{h} = \lim_{h \to 0^-} \frac{[1+h] - [1]}{h}$$
$$= \lim_{h \to 0^-} \frac{0-1}{h} = \lim_{h \to 0^-} \frac{-1}{h} = \infty$$

Consider the right hand limit of f at $x = 1$

$$\lim_{h \to 0^+} \frac{f(1+h) - f(1)}{h} = \lim_{h \to 0^+} \frac{[1+h] - [1]}{h}$$
$$= \lim_{h \to 0^+} \frac{1-1}{h} = \lim_{h \to 0^+} 0 = 0$$

Since the left and right hand limits of f at $x = 1$ are not equal, f is not differentiable at

$x = 1$

To check the differentiability of the given function at $x = 2$, consider the left hand limit

of f at $x = 2$

$$\lim_{h \to 0^-} \frac{f(2+h) - f(2)}{h} = \lim_{h \to 0^-} \frac{[2+h] - [2]}{h}$$
$$= \lim_{h \to 0^-} \frac{1-2}{h} = \lim_{h \to 0^-} \frac{-1}{h} = \infty$$

Consider the right hand limit of f at $x = 1$

$$\lim_{h \to 0^+} \frac{f(2+h) - f(2)}{h} = \lim_{h \to 0^+} \frac{[2+h] - [2]}{h}$$
$$= \lim_{h \to 0^+} \frac{2-2}{h} = \lim_{h \to 0^+} 0 = 0$$

Since the left and right hand limits of f at $x = 2$ are not equal, f is not differentiable at $x = 2$