

CLASS XII DETERMINANTS CHAPTER 4

EX. 4.4 SOLUTIONS

Question 1:

Write Minors and Cofactors of the elements of following determinants:

$$(i) \begin{vmatrix} 2 & -4 \\ 0 & 3 \end{vmatrix} \quad (ii) \begin{vmatrix} a & c \\ b & d \end{vmatrix}$$

ANS :

$$(i) \text{ The given determinant is } \begin{vmatrix} 2 & -4 \\ 0 & 3 \end{vmatrix}.$$

Minor of element a_{ij} is M_{ij} .

$$\therefore M_{11} = \text{minor of element } a_{11} = 3$$

$$M_{12} = \text{minor of element } a_{12} = 0$$

$$M_{21} = \text{minor of element } a_{21} = -4$$

$$M_{22} = \text{minor of element } a_{22} = 2$$

Cofactor of a_{ij} is $A_{ij} = (-1)^{i+j} M_{ij}$.

$$\therefore A_{11} = (-1)^{1+1} M_{11} = (-1)^2 (3) = 3$$

$$A_{12} = (-1)^{1+2} M_{12} = (-1)^3 (0) = 0$$

$$A_{21} = (-1)^{2+1} M_{21} = (-1)^3 (-4) = 4$$

$$A_{22} = (-1)^{2+2} M_{22} = (-1)^4 (2) = 2$$

$$(ii) \text{ The given determinant is } \begin{vmatrix} a & c \\ b & d \end{vmatrix}.$$

Minor of element a_{ij} is M_{ij} .

$$\therefore M_{11} = \text{minor of element } a_{11} = d$$

$$M_{12} = \text{minor of element } a_{12} = b$$

$$M_{21} = \text{minor of element } a_{21} = c$$

$$M_{22} = \text{minor of element } a_{22} = a$$

$$\text{Cofactor of } a_{ij} \text{ is } A_{ij} = (-1)^{i+j} M_{ij}$$

$$\therefore A_{11} = (-1)^{1+1} M_{11} = (-1)^2 (d) = d$$

$$A_{12} = (-1)^{1+2} M_{12} = (-1)^3 (b) = -b$$

$$A_{21} = (-1)^{2+1} M_{21} = (-1)^3 (c) = -c$$

$$A_{22} = (-1)^{2+2} M_{22} = (-1)^4 (a) = a$$

Question 2:

$$\text{(i) } \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} \quad \text{(ii) } \begin{vmatrix} 1 & 0 & 4 \\ 3 & 5 & -1 \\ 0 & 1 & 2 \end{vmatrix}$$

ANS :

(i) The given determinant is $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$.

By the definition of minors and cofactors, we have:

$$M_{11} = \text{minor of } a_{11} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$$

$$M_{12} = \text{minor of } a_{12} = \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} = 0$$

$$M_{13} = \text{minor of } a_{13} = \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} = 0$$

$$M_{21} = \text{minor of } a_{21} = \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} = 0$$

$$M_{22} = \text{minor of } a_{22} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$$

$$M_{23} = \text{minor of } a_{23} = \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} = 0$$

$$M_{31} = \text{minor of } a_{31} = \begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix} = 0$$

$$M_{32} = \text{minor of } a_{32} = \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} = 0$$

$$M_{33} = \text{minor of } a_{33} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$$

$$A_{11} = \text{cofactor of } a_{11} = (-1)^{1+1} M_{11} = 1$$

$$A_{12} = \text{cofactor of } a_{12} = (-1)^{1+2} M_{12} = 0$$

$$A_{13} = \text{cofactor of } a_{13} = (-1)^{1+3} M_{13} = 0$$

$$A_{21} = \text{cofactor of } a_{21} = (-1)^{2+1} M_{21} = 0$$

$$A_{22} = \text{cofactor of } a_{22} = (-1)^{2+2} M_{22} = 1$$

$$A_{23} = \text{cofactor of } a_{23} = (-1)^{2+3} M_{23} = 0$$

$$A_{31} = \text{cofactor of } a_{31} = (-1)^{3+1} M_{31} = 0$$

$$A_{32} = \text{cofactor of } a_{32} = (-1)^{3+2} M_{32} = 0$$

$$A_{33} = \text{cofactor of } a_{33} = (-1)^{3+3} M_{33} = 1$$

(ii) The given determinant is $\begin{vmatrix} 1 & 0 & 4 \\ 3 & 5 & -1 \\ 0 & 1 & 2 \end{vmatrix}$.

By definition of minors and cofactors, we have:

$$M_{11} = \text{minor of } a_{11} = \begin{vmatrix} 5 & -1 \\ 1 & 2 \end{vmatrix} = 10 + 1 = 11$$

$$M_{12} = \text{minor of } a_{12} = \begin{vmatrix} 3 & -1 \\ 0 & 2 \end{vmatrix} = 6 - 0 = 6$$

$$M_{13} = \text{minor of } a_{13} = \begin{vmatrix} 3 & 5 \\ 0 & 1 \end{vmatrix} = 3 - 0 = 3$$

$$M_{21} = \text{minor of } a_{21} = \begin{vmatrix} 0 & 4 \\ 1 & 2 \end{vmatrix} = 0 - 4 = -4$$

$$M_{22} = \text{minor of } a_{22} = \begin{vmatrix} 1 & 4 \\ 0 & 2 \end{vmatrix} = 2 - 0 = 2$$

$$M_{23} = \text{minor of } a_{23} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 - 0 = 1$$

$$M_{31} = \text{minor of } a_{31} = \begin{vmatrix} 0 & 4 \\ 5 & -1 \end{vmatrix} = 0 - 20 = -20$$

$$M_{32} = \text{minor of } a_{32} = \begin{vmatrix} 1 & 4 \\ 3 & -1 \end{vmatrix} = -1 - 12 = -13$$

$$M_{33} = \text{minor of } a_{33} = \begin{vmatrix} 1 & 0 \\ 3 & 5 \end{vmatrix} = 5 - 0 = 5$$

$$A_{11} = \text{cofactor of } a_{11} = (-1)^{1+1} M_{11} = 11$$

$$A_{12} = \text{cofactor of } a_{12} = (-1)^{1+2} M_{12} = -6$$

$$A_{13} = \text{cofactor of } a_{13} = (-1)^{1+3} M_{13} = 3$$

$$A_{21} = \text{cofactor of } a_{21} = (-1)^{2+1} M_{21} = 4$$

$$A_{22} = \text{cofactor of } a_{22} = (-1)^{2+2} M_{22} = 2$$

$$A_{23} = \text{cofactor of } a_{23} = (-1)^{2+3} M_{23} = -1$$

$$A_{31} = \text{cofactor of } a_{31} = (-1)^{3+1} M_{31} = -20$$

$$A_{32} = \text{cofactor of } a_{32} = (-1)^{3+2} M_{32} = 13$$

$$A_{33} = \text{cofactor of } a_{33} = (-1)^{3+3} M_{33} = 5$$

Question 3:

Using Cofactors of elements of second row, evaluate $\Delta = \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$.

ANS:

We have:

$$M_{21} = \begin{vmatrix} 3 & 8 \\ 2 & 3 \end{vmatrix} = 9 - 16 = -7$$

$$\therefore A_{21} = \text{cofactor of } a_{21} = (-1)^{2+1} M_{21} = 7$$

$$M_{22} = \begin{vmatrix} 5 & 8 \\ 1 & 3 \end{vmatrix} = 15 - 8 = 7$$

$$\therefore A_{22} = \text{cofactor of } a_{22} = (-1)^{2+2} M_{22} = 7$$

$$M_{23} = \begin{vmatrix} 5 & 3 \\ 1 & 2 \end{vmatrix} = 10 - 3 = 7$$

$$\therefore A_{23} = \text{cofactor of } a_{23} = (-1)^{2+3} M_{23} = -7$$

We know that Δ is equal to the sum of the product of the elements of the second row with their corresponding cofactors.

$$\therefore \Delta = a_{21}A_{21} + a_{22}A_{22} + a_{23}A_{23} = 2(7) + 0(7) + 1(-7) = 14 - 7 = 7$$

Question 4:

Using Cofactors of elements of third column, evaluate $\Delta = \begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix}$

ANS:

We have:

$$M_{13} = \begin{vmatrix} 1 & y \\ 1 & z \end{vmatrix} = z - y$$

$$M_{23} = \begin{vmatrix} 1 & x \\ 1 & z \end{vmatrix} = z - x$$

$$M_{33} = \begin{vmatrix} 1 & x \\ 1 & y \end{vmatrix} = y - x$$

$$\therefore A_{13} = \text{cofactor of } a_{13} = (-1)^{1+3} M_{13} = (z - y)$$

$$A_{23} = \text{cofactor of } a_{23} = (-1)^{2+3} M_{23} = -(z - x) = (x - z)$$

$$A_{33} = \text{cofactor of } a_{33} = (-1)^{3+3} M_{33} = (y - x)$$

We know that Δ is equal to the sum of the product of the elements of the second row with their corresponding cofactors.

$$\begin{aligned}
\therefore \Delta &= a_{13}A_{13} + a_{23}A_{23} + a_{33}A_{33} \\
&= yz(z-y) + zx(x-z) + xy(y-x) \\
&= yz^2 - y^2z + x^2z - xz^2 + xy^2 - x^2y \\
&= (x^2z - y^2z) + (yz^2 - xz^2) + (xy^2 - x^2y) \\
&= z(x^2 - y^2) + z^2(y-x) + xy(y-x) \\
&= z(x-y)(x+y) + z^2(y-x) + xy(y-x) \\
&= (x-y)[zx + zy - z^2 - xy] \\
&= (x-y)[z(x-z) + y(z-x)] \\
&= (x-y)(z-x)[-z+y] \\
&= (x-y)(y-z)(z-x)
\end{aligned}$$

Hence, $\Delta = (x-y)(y-z)(z-x)$.

Question 5:

If $\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ and A_{ij} is Cofactors of a_{ij} , then value of Δ is given by

- (A) $a_{11}A_{31} + a_{12}A_{32} + a_{13}A_{33}$ (B) $a_{11}A_{11} + a_{12}A_{21} + a_{13}A_{31}$
(C) $a_{21}A_{11} + a_{22}A_{12} + a_{23}A_{13}$ (D) $a_{11}A_{11} + a_{21}A_{21} + a_{31}A_{31}$

ANS:

We know that:

$\Delta =$ Sum of the product of the elements of a column (or a row) with their corresponding cofactors

$$\therefore \Delta = a_{11}A_{11} + a_{21}A_{21} + a_{31}A_{31}$$

Hence, the value of Δ is given by the expression given in alternative D.

The correct answer is D.

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