

CLASS XII MATRICES CHAPTER 3

EX. 3.4 SOLUTIONS

Question 1:

Find the inverse of each of the matrices, if it exists.

$$\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$$

$$\text{Let } A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$$

We know that $A = IA$

$$\therefore \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 1 & -1 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} A \quad (\mathbf{R_2 \rightarrow R_2 - 2R_1})$$

$$\Rightarrow \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix} A \quad (\mathbf{R_2 \rightarrow \frac{1}{5}R_2})$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{3}{5} & \frac{1}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix} A \quad (\mathbf{R_1 \rightarrow R_1 + R_2})$$

$$\therefore A^{-1} = \begin{bmatrix} \frac{3}{5} & \frac{1}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix}$$

Question 2:

Find the inverse of each of the matrices, if it exists.

$$\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$$

$$\text{Let } A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$$

We know that $A = IA$

$$\therefore \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} A \quad (\mathbf{R_1 \rightarrow R_1 - R_2})$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} A \quad (\mathbf{R_2 \rightarrow R_2 - R_1})$$

$$\therefore A^{-1} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$$

Question 3:

Find the inverse of each of the matrices, if it exists.

$$\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$$

$$\text{Let } A = \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$$

We know that $A = IA$

$$\therefore \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} A \quad (\text{R}_2 \rightarrow \text{R}_2 - 2\text{R}_1)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix} A \quad (\text{R}_1 \rightarrow \text{R}_1 - 3\text{R}_2)$$

$$\therefore A^{-1} = \begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix}$$

Question 4:

Find the inverse of each of the matrices, if it exists.

$$\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$$

$$\text{Let } A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$$

We know that $A = LA$

$$\therefore \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 1 & \frac{3}{2} \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{bmatrix} A \quad \left(R_1 \rightarrow \frac{1}{2}R_1 \right)$$

$$\Rightarrow \begin{bmatrix} 1 & \frac{3}{2} \\ 0 & -\frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ -\frac{5}{2} & 1 \end{bmatrix} A \quad (R_2 \rightarrow R_2 - 5R_1)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & -\frac{1}{2} \end{bmatrix} = \begin{bmatrix} -7 & 3 \\ -\frac{5}{2} & 1 \end{bmatrix} A \quad (R_1 \rightarrow R_1 + 3R_2)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -7 & 3 \\ 5 & -2 \end{bmatrix} A \quad (R_2 \rightarrow -2R_1)$$

$$\therefore A^{-1} = \begin{bmatrix} -7 & 3 \\ 5 & -2 \end{bmatrix}$$

Question 5:

Find the inverse of each of the matrices, if it exists.

$$\begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix}$$

$$\text{Let } A = \begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix}$$

We know that $A = IA$

$$\therefore \begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 1 & \frac{1}{2} \\ 7 & 4 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{bmatrix} A \quad \left(R_1 \rightarrow \frac{1}{2}R_1 \right)$$

$$\Rightarrow \begin{bmatrix} 1 & \frac{1}{2} \\ 0 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ -\frac{7}{2} & 1 \end{bmatrix} A \quad (R_2 \rightarrow R_2 - 7R_1)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ -\frac{7}{2} & 1 \end{bmatrix} A \quad (R_1 \rightarrow R_1 - R_2)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix} A \quad (R_2 \rightarrow 2R_2)$$

$$\therefore A^{-1} = \begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix}$$

Question 6:

Find the inverse of each of the matrices, if it exists.

$$\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$$

$$\text{Let } A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$$

We know that $A = IA$

$$\therefore \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 1 & \frac{5}{2} \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{bmatrix} A \quad \left(R_1 \rightarrow \frac{1}{2}R_1 \right)$$

$$\Rightarrow \begin{bmatrix} 1 & \frac{5}{2} \\ 0 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ -\frac{1}{2} & 1 \end{bmatrix} A \quad (R_2 \rightarrow R_2 - R_1)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 3 & -5 \\ -\frac{1}{2} & 1 \end{bmatrix} A \quad (R_1 \rightarrow R_2 - 5R_2)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix} A \quad (R_2 \rightarrow 2R_2)$$

$$\therefore A^{-1} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$$

Question 7:

Find the inverse of each of the matrices, if it exists.

$$\begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$$

$$\text{Let } A = \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$$

We know that $A = AI$

$$\therefore \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix} = A \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} = A \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} \quad (C_1 \rightarrow C_1 - 2C_2)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = A \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix} \quad (C_2 \rightarrow C_2 - C_1)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = A \begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix} \quad (C_1 \rightarrow C_1 - C_2)$$

$$\therefore A^{-1} = \begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}$$

Question 8:

Find the inverse of each of the matrices, if it exists.

$$\begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix}$$

$$\text{Let } A = \begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix}$$

We know that $A = IA$

$$\therefore \begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} A \quad (\text{R}_1 \rightarrow \text{R}_1 - \text{R}_2)$$

$$\Rightarrow \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -3 & 4 \end{bmatrix} A \quad (\text{R}_2 \rightarrow \text{R}_2 - 3\text{R}_1)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & -5 \\ -3 & 4 \end{bmatrix} A \quad (\text{R}_1 \rightarrow \text{R}_1 - \text{R}_2)$$

$$\therefore A^{-1} = \begin{bmatrix} 4 & -5 \\ -3 & 4 \end{bmatrix}$$

Question 10:

Find the inverse of each of the matrices, if it exists.

$$\begin{bmatrix} 3 & -1 \\ -4 & 2 \end{bmatrix}$$

$$\text{Let } A = \begin{bmatrix} 3 & -1 \\ -4 & 2 \end{bmatrix}$$

We know that $A = AI$

$$\therefore \begin{bmatrix} 3 & -1 \\ -4 & 2 \end{bmatrix} = A \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = A \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \quad (C_1 \rightarrow C_1 + 2C_2)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} = A \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix} \quad (C_2 \rightarrow C_2 + C_1)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = A \begin{bmatrix} 1 & \frac{1}{2} \\ 2 & \frac{3}{2} \end{bmatrix} \quad \left(C_2 \rightarrow \frac{1}{2}C_2\right)$$

$$\therefore A^{-1} = \begin{bmatrix} 1 & \frac{1}{2} \\ 2 & \frac{3}{2} \end{bmatrix}$$

Question 11:

Find the inverse of each of the matrices, if it exists.

$$\begin{bmatrix} 2 & -6 \\ 1 & -2 \end{bmatrix}$$

$$\text{Let } A = \begin{bmatrix} 2 & -6 \\ 1 & -2 \end{bmatrix}$$

We know that $A = AI$

$$\therefore \begin{bmatrix} 2 & -6 \\ 1 & -2 \end{bmatrix} = A \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix} = A \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} \quad (C_2 \rightarrow C_2 + 3C_1)$$

$$\Rightarrow \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} = A \begin{bmatrix} -2 & 3 \\ -1 & 1 \end{bmatrix} \quad (C_1 \rightarrow C_1 - C_2)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = A \begin{bmatrix} -1 & 3 \\ -\frac{1}{2} & 1 \end{bmatrix} \quad \left(C_1 \rightarrow \frac{1}{2}C_1\right)$$

$$\therefore A^{-1} = \begin{bmatrix} -1 & 3 \\ -\frac{1}{2} & 1 \end{bmatrix}$$

Question 12:

Find the inverse of each of the matrices, if it exists.

$$\begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix}$$

$$\text{Let } A = \begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix}$$

We know that $A = LA$

$$\therefore \begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 1 & -\frac{1}{2} \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{6} & 0 \\ 0 & 1 \end{bmatrix} A \quad \left(R_1 \rightarrow \frac{1}{6} R_1 \right)$$

$$\Rightarrow \begin{bmatrix} 1 & -\frac{1}{2} \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{6} & 0 \\ \frac{1}{3} & 1 \end{bmatrix} A \quad (R_2 \rightarrow R_2 + 2R_1)$$

Now, in the above equation, we can see all the zeros in the second row of the matrix on the L.H.S.

Therefore, A^{-1} does not exist.

Question 13:

Find the inverse of each of the matrices, if it exists.

$$\begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$$

$$\text{Let } A = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$$

We know that $A = LA$

$$\begin{aligned} \therefore \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix} &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A \\ \Rightarrow \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} &= \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} A && (R_1 \rightarrow R_1 + R_2) \\ \Rightarrow \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} &= \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} A && (R_2 \rightarrow R_2 + R_1) \\ \Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} &= \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} A && (R_1 \rightarrow R_1 + R_2) \end{aligned}$$

$$\therefore A^{-1} = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

Question 14:

Find the inverse of each of the matrices, if it exists.

$$\begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$$

$$\text{Let } A = \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$$

We know that $A = IA$

$$\therefore \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_1 \rightarrow R_1 - \frac{1}{2}R_2$, we have:

$$\begin{bmatrix} 0 & 0 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 1 & -\frac{1}{2} \\ 0 & 1 \end{bmatrix} A$$

Now, in the above equation, we can see all the zeros in the first row of the matrix on the L.H.S.

Therefore, A^{-1} does not exist.

Question 15:

Find the inverse of each of the matrices, if it exists.

$$\begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix}$$

$$\text{Let } A = \begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix}$$

We know that $A = IA$

$$\therefore \begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 2 & -3 & 3 \\ 0 & 5 & 0 \\ 3 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \quad (R_2 \rightarrow R_2 - R_1)$$

$$\Rightarrow \begin{bmatrix} 2 & -3 & 3 \\ 0 & 1 & 0 \\ 3 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{5} & \frac{1}{5} & 0 \\ 0 & 0 & 1 \end{bmatrix} A \quad (R_2 \rightarrow \frac{1}{5})$$

$$\Rightarrow \begin{bmatrix} -1 & -1 & 1 \\ 0 & 1 & 0 \\ 3 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ -\frac{1}{5} & \frac{1}{5} & 0 \\ 0 & 0 & 1 \end{bmatrix} A \quad (R_1 \rightarrow R_1 - R_3)$$

$$\Rightarrow \begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & 0 \\ 3 & 0 & 2 \end{bmatrix} = \begin{bmatrix} \frac{4}{5} & \frac{1}{5} & -1 \\ -\frac{1}{5} & \frac{1}{5} & 0 \\ -\frac{2}{5} & \frac{2}{5} & 1 \end{bmatrix} A \quad (R_1 \rightarrow R_1 + R_2 \text{ and } R_3 \rightarrow R_3 + 2R_2)$$

$$\Rightarrow \begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 5 \end{bmatrix} = \begin{bmatrix} \frac{4}{5} & \frac{1}{5} & -1 \\ -\frac{1}{5} & \frac{1}{5} & 0 \\ 2 & 1 & -2 \end{bmatrix} A \quad (\mathbf{R}_3 \rightarrow \mathbf{R}_3 + 3\mathbf{R}_1)$$

$$\Rightarrow \begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{4}{5} & \frac{1}{5} & -1 \\ -\frac{1}{5} & \frac{1}{5} & 0 \\ \frac{2}{5} & \frac{1}{5} & -\frac{2}{5} \end{bmatrix} A \quad \left(\mathbf{R}_3 \rightarrow \frac{1}{5}\mathbf{R}_3\right)$$

$$\Rightarrow \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{2}{5} & 0 & -\frac{3}{5} \\ -\frac{1}{5} & \frac{1}{5} & 0 \\ \frac{2}{5} & \frac{1}{5} & -\frac{2}{5} \end{bmatrix} A \quad (\mathbf{R}_1 \rightarrow \mathbf{R}_1 - \mathbf{R}_3)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -\frac{2}{5} & 0 & \frac{3}{5} \\ -\frac{1}{5} & \frac{1}{5} & 0 \\ \frac{2}{5} & \frac{1}{5} & -\frac{2}{5} \end{bmatrix} A \quad (\mathbf{R}_1 \rightarrow (-1)\mathbf{R}_1)$$

$$\therefore A^{-1} = \begin{bmatrix} -\frac{2}{5} & 0 & \frac{3}{5} \\ -\frac{1}{5} & \frac{1}{5} & 0 \\ \frac{2}{5} & \frac{1}{5} & -\frac{2}{5} \end{bmatrix}$$

Question 16:

Find the inverse of each of the matrices, if it exists.

$$\begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix}$$

$$\text{Let } A = \begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix}$$

We know that $A = IA$

$$\therefore \begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \rightarrow R_2 + 3R_1$ and $R_3 \rightarrow R_3 - 2R_1$, we have:

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & 9 & -11 \\ 0 & -1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} A$$

Applying $R_1 \rightarrow R_1 + 3R_3$ and $R_2 \rightarrow R_2 + 8R_3$, we have:

$$\begin{bmatrix} 1 & 0 & 10 \\ 0 & 1 & 21 \\ 0 & -1 & 4 \end{bmatrix} = \begin{bmatrix} -5 & 0 & 3 \\ -13 & 1 & 8 \\ -2 & 0 & 1 \end{bmatrix} A$$

Applying $R_3 \rightarrow R_3 + R_2$, we have:

$$\begin{bmatrix} 1 & 0 & 10 \\ 0 & 1 & 21 \\ 0 & 0 & 25 \end{bmatrix} = \begin{bmatrix} -5 & 0 & 3 \\ -13 & 1 & 8 \\ -15 & 1 & 9 \end{bmatrix} A$$

Applying $R_3 \rightarrow \frac{1}{25}R_3$, we have:

$$\begin{bmatrix} 1 & 0 & 10 \\ 0 & 1 & 21 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -5 & 0 & 3 \\ -13 & 1 & 8 \\ -\frac{3}{5} & \frac{1}{25} & \frac{9}{25} \end{bmatrix} A$$

Applying $R_1 \rightarrow R_1 - 10R_3$, and $R_2 \rightarrow R_2 - 21R_3$, we have:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -\frac{2}{5} & -\frac{3}{5} \\ -\frac{2}{5} & \frac{4}{25} & \frac{11}{25} \\ -\frac{3}{5} & \frac{1}{25} & \frac{9}{25} \end{bmatrix} A$$

$$\therefore A^{-1} = \begin{bmatrix} 1 & -\frac{2}{5} & -\frac{3}{5} \\ -\frac{2}{5} & \frac{4}{25} & \frac{11}{25} \\ -\frac{3}{5} & \frac{1}{25} & \frac{9}{25} \end{bmatrix}$$

Question 17:

Find the inverse of each of the matrices, if it exists.

$$\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

$$\text{Let } A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

We know that $A = LA$

$$\therefore \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_1 \rightarrow \frac{1}{2}R_1$, we have:

$$\begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \rightarrow R_2 - 5R_1$, we have:

$$\begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{5}{2} \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -\frac{5}{2} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_3 \rightarrow R_3 - R_2$, we have:

$$\begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{5}{2} \\ 0 & 0 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -\frac{5}{2} & 1 & 0 \\ \frac{5}{2} & -1 & 1 \end{bmatrix} A$$

Applying $R_3 \rightarrow 2R_3$, we have:

$$\begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{5}{2} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -\frac{5}{2} & 1 & 0 \\ 5 & -2 & 2 \end{bmatrix} A$$

Applying $R_1 \rightarrow R_1 + \frac{1}{2}R_3$, and $R_2 \rightarrow R_2 - \frac{5}{2}R_3$, we have:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} A$$

$$\therefore A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

Question 18:

Matrices A and B will be inverse of each other only if

A. $AB = BA$

C. $AB = 0, BA = I$

B. $AB = BA = 0$

D. $AB = BA = I$

Answer: D

We know that if A is a square matrix of order m , and if there exists another square matrix B of the same order m , such that $AB = BA = I$, then B is said to be the inverse of A . In this case, it is clear that A is the inverse of B .

Thus, matrices A and B will be inverses of each other only if $AB = BA = I$.

