

## CLASS XII CHAPTER 1 RELATION AND FUNCTION

### NCERT EX 1.2 SOLUTIONS

1. Show that the function  $f: \mathbf{R}_* \rightarrow \mathbf{R}_*$  defined by  $f(x) = \frac{1}{x}$  is one-one and onto, where  $\mathbf{R}_*$  is the set of all non-zero real numbers. Is the result true, if the domain  $\mathbf{R}_*$  is replaced by  $\mathbf{N}$  with co-domain being same as  $\mathbf{R}_*$ ?

**ANS:**

It is given that  $f: \mathbf{R}_* \rightarrow \mathbf{R}_*$  is defined by  $f(x) = \frac{1}{x}$ .

One-one:

$$f(x) = f(y)$$

$$\Rightarrow \frac{1}{x} = \frac{1}{y}$$

$$\Rightarrow x = y$$

$\therefore f$  is one-one.

Onto:

It is clear that for  $y \in \mathbf{R}_*$ , there exists  $x = \frac{1}{y} \in \mathbf{R}_*$  (Exists as  $y \neq 0$ ) such that

$$f(x) = \frac{1}{\left(\frac{1}{y}\right)} = y.$$

$\therefore f$  is onto.

Thus, the given function ( $f$ ) is one-one and onto.

Now, consider function  $g: \mathbf{N} \rightarrow \mathbf{R}_*$  defined by

$$g(x) = \frac{1}{x}.$$

We have,

$$g(x_1) = g(x_2) \Rightarrow \frac{1}{x_1} = \frac{1}{x_2} \Rightarrow x_1 = x_2$$

$\therefore g$  is one-one.

Further, it is clear that  $g$  is not onto as for  $1.2 \in \mathbf{R}_*$  there does not exist any  $x$  in  $\mathbf{N}$  such that  $g(x) = \frac{1}{1.2}$ .

Hence, function  $g$  is one-one but not onto.

2. Check the injectivity and surjectivity of the following functions:

(i)  $f: \mathbf{N} \rightarrow \mathbf{N}$  given by  $f(x) = x^2$

(ii)  $f: \mathbf{Z} \rightarrow \mathbf{Z}$  given by  $f(x) = x^2$

(iii)  $f: \mathbf{R} \rightarrow \mathbf{R}$  given by  $f(x) = x^2$

(iv)  $f: \mathbf{N} \rightarrow \mathbf{N}$  given by  $f(x) = x^3$

(v)  $f: \mathbf{Z} \rightarrow \mathbf{Z}$  given by  $f(x) = x^3$

**ANS:**

(i)  $f: \mathbf{N} \rightarrow \mathbf{N}$  is given by,

$$f(x) = x^2$$

It is seen that for  $x, y \in \mathbf{N}$ ,  $f(x) = f(y) \Rightarrow x^2 = y^2 \Rightarrow x = y$ .

$\therefore f$  is injective.

Now,  $2 \in \mathbf{N}$ . But, there does not exist any  $x$  in  $\mathbf{N}$  such that  $f(x) = x^2 = 2$ .

$\therefore f$  is not surjective.

Hence, function  $f$  is injective but not surjective.

(ii)  $f: \mathbf{Z} \rightarrow \mathbf{Z}$  is given by,

$$f(x) = x^2$$

It is seen that  $f(-1) = f(1) = 1$ , but  $-1 \neq 1$ .

$\therefore f$  is not injective.

Now,  $-2 \in \mathbf{Z}$ . But, there does not exist any element  $x \in \mathbf{Z}$  such that  $f(x) = x^2 = -2$ .

$\therefore f$  is not surjective.

Hence, function  $f$  is neither injective nor surjective.

(iii)  $f: \mathbf{R} \rightarrow \mathbf{R}$  is given by,

$$f(x) = x^2$$

It is seen that  $f(-1) = f(1) = 1$ , but  $-1 \neq 1$ .

$\therefore f$  is not injective.

Now,  $-2 \in \mathbf{R}$ . But, there does not exist any element  $x \in \mathbf{R}$  such that  $f(x) = x^2 = -2$ .

$\therefore f$  is not surjective.

Hence, function  $f$  is neither injective nor surjective.

(iv)  $f: \mathbf{N} \rightarrow \mathbf{N}$  given by,

$$f(x) = x^3$$

It is seen that for  $x, y \in \mathbf{N}$ ,  $f(x) = f(y) \Rightarrow x^3 = y^3 \Rightarrow x = y$ .

$\therefore f$  is injective.

Now,  $2 \in \mathbf{N}$ . But, there does not exist any element  $x$  in domain  $\mathbf{N}$  such that  $f(x) = x^3 = 2$ .

$\therefore f$  is not surjective

Hence, function  $f$  is injective but not surjective.

(v)  $f: \mathbf{Z} \rightarrow \mathbf{Z}$  is given by,

$$f(x) = x^3$$

It is seen that for  $x, y \in \mathbf{Z}$ ,  $f(x) = f(y) \Rightarrow x^3 = y^3 \Rightarrow x = y$ .

$\therefore f$  is injective.

Now,  $2 \in \mathbf{Z}$ . But, there does not exist any element  $x$  in domain  $\mathbf{Z}$  such that  $f(x) = x^3 = 2$ .

$\therefore f$  is not surjective.

Hence, function  $f$  is injective but not surjective.

- 3. Prove that the Greatest Integer Function  $f: \mathbf{R} \rightarrow \mathbf{R}$ , given by  $f(x) = [x]$ , is neither one-one nor onto, where  $[x]$  denotes the greatest integer less than or equal to  $x$ .**

**ANS:**

$f: \mathbf{R} \rightarrow \mathbf{R}$  is given by,

$$f(x) = [x]$$

It is seen that  $f(1.2) = [1.2] = 1$ ,  $f(1.9) = [1.9] = 1$ .

$\therefore f(1.2) = f(1.9)$ , but  $1.2 \neq 1.9$ .

$\therefore f$  is not one-one.

Now, consider  $0.7 \in \mathbf{R}$ .

It is known that  $f(x) = [x]$  is always an integer. Thus, there does not exist any element  $x \in \mathbf{R}$  such that  $f(x) = 0.7$ .

$\therefore f$  is not onto.

Hence, the greatest integer function is neither one-one nor onto.

4. Show that the Modulus Function  $f: \mathbf{R} \rightarrow \mathbf{R}$ , given by  $f(x) = |x|$ , is neither one-one nor onto, where  $|x|$  is  $x$ , if  $x$  is positive or 0 and  $|x|$  is  $-x$ , if  $x$  is negative.

**ANS:**

$f: \mathbf{R} \rightarrow \mathbf{R}$  is given by,

$$f(x) = |x| = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$$

It is seen that  $f(-1) = |-1| = 1$ ,  $f(1) = |1| = 1$ .

$\therefore f(-1) = f(1)$ , but  $-1 \neq 1$ .

$\therefore f$  is not one-one.

Now, consider  $-1 \in \mathbf{R}$ .

It is known that  $f(x) = |x|$  is always non-negative. Thus, there does not exist any element  $x$  in domain  $\mathbf{R}$  such that  $f(x) = |x| = -1$ .

$\therefore f$  is not onto.

Hence, the modulus function is neither one-one nor onto.

5. Show that the Signum Function  $f: \mathbf{R} \rightarrow \mathbf{R}$ , given by

$$f(x) = \begin{cases} 1, & \text{if } x > 0 \\ 0, & \text{if } x = 0 \\ -1, & \text{if } x < 0 \end{cases}$$

is neither one-one nor onto.

**ANS:**

It is seen that  $f(1) = f(2) = 1$ , but  $1 \neq 2$ .

$\therefore f$  is not one-one.

Now, as  $f(x)$  takes only 3 values (1, 0, or  $-1$ ) for the element  $-2$  in co-domain  $\mathbf{R}$ , there does not exist any  $x$  in domain  $\mathbf{R}$  such that  $f(x) = -2$ .

$\therefore f$  is not onto.

Hence, the signum function is neither one-one nor onto.

6. Let  $A = \{1, 2, 3\}$ ,  $B = \{4, 5, 6, 7\}$  and let  $f = \{(1, 4), (2, 5), (3, 6)\}$  be a function from  $A$  to  $B$ . Show that  $f$  is one-one.

**ANS:**

It is given that  $A = \{1, 2, 3\}$ ,  $B = \{4, 5, 6, 7\}$ .

$f: A \rightarrow B$  is defined as  $f = \{(1, 4), (2, 5), (3, 6)\}$ .

$$\therefore f(1) = 4, f(2) = 5, f(3) = 6$$

It is seen that the images of distinct elements of  $A$  under  $f$  are distinct.

Hence, function  $f$  is one-one.

7. In each of the following cases, state whether the function is one-one, onto or bijective. Justify your answer.

(i)  $f: \mathbf{R} \rightarrow \mathbf{R}$  defined by  $f(x) = 3 - 4x$

(ii)  $f: \mathbf{R} \rightarrow \mathbf{R}$  defined by  $f(x) = 1 + x^2$

**ANS:**

(i)  $f: \mathbf{R} \rightarrow \mathbf{R}$  is defined as  $f(x) = 3 - 4x$ .

Let  $x_1, x_2 \in \mathbf{R}$  such that  $f(x_1) = f(x_2)$ .

$$\Rightarrow 3 - 4x_1 = 3 - 4x_2$$

$$\Rightarrow -4x_1 = -4x_2$$

$$\Rightarrow x_1 = x_2$$

$\therefore f$  is one-one.

For any real number ( $y$ ) in  $\mathbf{R}$ , there exists  $\frac{3-y}{4}$  in  $\mathbf{R}$  such that

$$f\left(\frac{3-y}{4}\right) = 3 - 4\left(\frac{3-y}{4}\right) = y.$$

$\therefore f$  is onto.

Hence,  $f$  is bijective.

(ii)  $f: \mathbf{R} \rightarrow \mathbf{R}$  is defined as

$$f(x) = 1 + x^2.$$

Let  $x_1, x_2 \in \mathbf{R}$  such that  $f(x_1) = f(x_2)$ .

$$\Rightarrow 1 + x_1^2 = 1 + x_2^2$$

$$\Rightarrow x_1^2 = x_2^2$$

$$\Rightarrow x_1 = \pm x_2$$

$\therefore f(x_1) = f(x_2)$  does not imply that  $x_1 = x_2$ .

For instance,

$$f(1) = f(-1) = 2$$

$\therefore f$  is not one-one.

Consider an element  $-2$  in co-domain  $\mathbf{R}$ .

It is seen that  $f(x) = 1 + x^2$  is positive for all  $x \in \mathbf{R}$ .

Thus, there does not exist any  $x$  in domain  $\mathbf{R}$  such that  $f(x) = -2$ .

$\therefore f$  is not onto.

Hence,  $f$  is neither one-one nor onto.

- 8.** Let  $A$  and  $B$  be sets. Show that  $f: A \times B \rightarrow B \times A$  such that  $f(a, b) = (b, a)$  is bijective function.

**ANS:**

$f: A \times B \rightarrow B \times A$  is defined as  $f(a, b) = (b, a)$ .

Let  $(a_1, b_1), (a_2, b_2) \in A \times B$  such that  $f(a_1, b_1) = f(a_2, b_2)$ .

$$\Rightarrow (b_1, a_1) = (b_2, a_2)$$

$$\Rightarrow b_1 = b_2 \text{ and } a_1 = a_2$$

$$\Rightarrow (a_1, b_1) = (a_2, b_2)$$

$\therefore f$  is one-one.

Now, let  $(b, a) \in B \times A$  be any element.

Then, there exists  $(a, b) \in A \times B$  such that  $f(a, b) = (b, a)$ . [By definition of  $f$ ]

$\therefore f$  is onto.

Hence,  $f$  is bijective.

9. Let  $f: \mathbb{N} \rightarrow \mathbb{N}$  be defined by  $f(n) = \begin{cases} \frac{n+1}{2}, & \text{if } n \text{ is odd} \\ \frac{n}{2}, & \text{if } n \text{ is even} \end{cases}$  for all  $n \in \mathbb{N}$ .

**ANS:**

It can be observed that:

$$f(1) = \frac{1+1}{2} = 1 \text{ and } f(2) = \frac{2}{2} = 1 \quad [\text{By definition of } f]$$

$$\therefore f(1) = f(2), \text{ where } 1 \neq 2.$$

$\therefore f$  is not one-one.

Consider a natural number  $(n)$  in co-domain  $\mathbb{N}$ .

Case I:  $n$  is odd

$\therefore n = 2r + 1$  for some  $r \in \mathbb{N}$ . Then, there exists  $4r + 1 \in \mathbb{N}$  such that

$$f(4r+1) = \frac{4r+1+1}{2} = 2r+1.$$

Case II:  $n$  is even

$\therefore n = 2r$  for some  $r \in \mathbb{N}$ . Then, there exists  $4r \in \mathbb{N}$  such that  $f(4r) = \frac{4r}{2} = 2r$ .

$\therefore f$  is onto.

Hence,  $f$  is not a bijective function.

10. Let  $A = \mathbb{R} - \{3\}$  and  $B = \mathbb{R} - \{1\}$ . Consider the function  $f: A \rightarrow B$  defined by

$$f(x) = \left( \frac{x-2}{x-3} \right). \text{ Is } f \text{ one-one and onto? Justify your answer.}$$

**ANS:**

Let  $x, y \in A$  such that  $f(x) = f(y)$ .

$$\Rightarrow \frac{x-2}{x-3} = \frac{y-2}{y-3}$$

$$\Rightarrow (x-2)(y-3) = (y-2)(x-3)$$

$$\Rightarrow xy - 3x - 2y + 6 = xy - 3y - 2x + 6$$

$$\Rightarrow -3x - 2y = -3y - 2x$$

$$\Rightarrow 3x - 2x = 3y - 2y$$

$$\Rightarrow x = y$$

$\therefore f$  is one-one.

Let  $y \in B = \mathbb{R} - \{1\}$ . Then,  $y \neq 1$ .

The function  $f$  is onto if there exists  $x \in A$  such that  $f(x) = y$ .

Now,

$$\begin{aligned}f(x) &= y \\ \Rightarrow \frac{x-2}{x-3} &= y \\ \Rightarrow x-2 &= xy-3y \\ \Rightarrow x(1-y) &= -3y+2 \\ \Rightarrow x &= \frac{2-3y}{1-y} \in A \quad [y \neq 1]\end{aligned}$$

Thus, for any  $y \in B$ , there exists  $\frac{2-3y}{1-y} \in A$  such that

$$f\left(\frac{2-3y}{1-y}\right) = \frac{\left(\frac{2-3y}{1-y}\right)^{-2}}{\left(\frac{2-3y}{1-y}\right)^{-3}} = \frac{2-3y-2+2y}{2-3y-3+3y} = \frac{-y}{-1} = y.$$

$\therefore f$  is onto.

Hence, function  $f$  is one-one and onto.

**11.** Let  $f: \mathbf{R} \rightarrow \mathbf{R}$  be defined as  $f(x) = x^4$ . Choose the correct answer.

- (A)  $f$  is one-one onto                      (B)  $f$  is many-one onto  
(C)  $f$  is one-one but not onto            (D)  $f$  is neither one-one nor onto.

**ANS:**

$f: \mathbf{R} \rightarrow \mathbf{R}$  is defined as  $f(x) = x^4$ .

Let  $x, y \in \mathbf{R}$  such that  $f(x) = f(y)$ .

$$\begin{aligned}\Rightarrow x^4 &= y^4 \\ \Rightarrow x &= \pm y\end{aligned}$$

$\therefore f(x_1) = f(x_2)$  does not imply that  $x_1 = x_2$ .

For instance,

$$f(1) = f(-1) = 1$$

$\therefore f$  is not one-one.

Consider an element 2 in co-domain  $\mathbf{R}$ . It is clear that there does not exist any  $x$  in domain  $\mathbf{R}$  such that  $f(x) = 2$ .

$\therefore f$  is not onto.

Hence, function  $f$  is neither one-one nor onto.

The correct answer is D.



12. Let  $f: \mathbf{R} \rightarrow \mathbf{R}$  be defined as  $f(x) = 3x$ . Choose the correct answer.
- (A)  $f$  is one-one onto                      (B)  $f$  is many-one onto  
(C)  $f$  is one-one but not onto            (D)  $f$  is neither one-one nor onto.

**ANS:**

$f: \mathbf{R} \rightarrow \mathbf{R}$  is defined as  $f(x) = 3x$ .

Let  $x, y \in \mathbf{R}$  such that  $f(x) = f(y)$ .

$$\Rightarrow 3x = 3y$$

$$\Rightarrow x = y$$

$\therefore f$  is one-one.

Also, for any real number ( $y$ ) in co-domain  $\mathbf{R}$ , there exists  $\frac{y}{3}$  in  $\mathbf{R}$  such that  $f\left(\frac{y}{3}\right) = 3\left(\frac{y}{3}\right) = y$ .

$\therefore f$  is onto.

Hence, function  $f$  is one-one and onto.

The correct answer is A.