SOLUTION KEY CLASS XII C TEST (EX. 6.1 – 6.4) HELD ON 09-AUGUST-2011

Ans 1.

The volume of a cone (V) with radius (r) and height (h) is given by,

$$V = \frac{1}{3}\pi r^2 h$$

It is given that,

$$h = \frac{1}{6}r \Rightarrow r = 6h$$

$$\therefore V = \frac{1}{3}\pi(6h)^2 h = 12\pi h^3$$

The rate of change of volume with respect to time (t) is given by,

$$\frac{dV}{dt} = 12\pi \frac{d}{dh}\left(h^3\right)\cdot\frac{dh}{dt} \quad \text{[By chain rule]}$$

$$= 12\pi\left(3h^2\right)\frac{dh}{dt}$$

$$= 36\pi h^2 \frac{dh}{dt}$$

It is also given that $\frac{dV}{dt} = 12 \text{ cm}^3/\text{s}$.

Therefore, when $h = 4$ cm, we have:

$$12 = 36\pi(4)^2 \frac{dh}{dt}$$

$$\Rightarrow \frac{dh}{dt} = \frac{12}{36\pi(16)} = \frac{1}{48\pi}$$

Hence, when the height of the sand cone is 4 cm, its height is increasing at the rate of $\frac{1}{48\pi}$ cm/s.

Ans 2. Do yourself.

Ans 3.

The equation of the given curve is $y = x^3 + 2x + 6$.

The slope of the tangent to the given curve at any point (x, y) is given by,

$$\frac{dy}{dx} = 3x^2 + 2$$

$\therefore$ Slope of the normal to the given curve at any point (x, y)

$$= \frac{-1}{\text{Slope of the tangent at the point } (x, y)}$$
$$= \frac{-1}{3x^2 + 2}$$

The equation of the given line is $x + 14y + 4 = 0$.

$x + 14y + 4 = 0 \Rightarrow y = -\frac{1}{14}x - \frac{4}{14}$ (which is of the form $y = mx + c$)

$\therefore$Slope of the given line $= \frac{-1}{14}$

If the normal is parallel to the line, then we must have the slope of the normal being equal to the slope of the line.

$$\therefore \frac{-1}{3x^2 + 2} = \frac{-1}{14}$$
$$\Rightarrow 3x^2 + 2 = 14$$
$$\Rightarrow 3x^2 = 12$$
$$\Rightarrow x^2 = 4$$
$$\Rightarrow x = \pm 2$$

When $x = 2$, $y = 8 + 4 + 6 = 18$.

When $x = -2$, $y = -8 - 4 + 6 = -6$.

Therefore, there are two normals to the given curve with slope $\frac{-1}{14}$ and passing through the points (2, 18) and (−2, −6).

Thus, the equation of the normal through (2, 18) is given by,

$$y - 18 = \frac{-1}{14}(x - 2)$$
$$\Rightarrow 14y - 252 = -x + 2$$
$$\Rightarrow x + 14y - 254 = 0$$

And, the equation of the normal through (−2, −6) is given by,

$$y - (-6) = \frac{-1}{14}[x - (-2)]$$
$$\Rightarrow y + 6 = \frac{-1}{14}(x + 2)$$
$$\Rightarrow 14y + 84 = -x - 2$$
$$\Rightarrow x + 14y + 86 = 0$$

Hence, the equations of the normals to the given curve (which are parallel to the given line) are $x + 14y - 254 = 0$ and $x + 14y + 86 = 0$.

Ans 4. Do yourself.