

SOLUTION KEY CLASS XII B TEST (EX. 6.1 – 6.4) HELD ON 09-AUGUST-2011

Ans 1. Do yourself.

Ans 2.

$$y = \log(1+x) - \frac{2x}{2+x}$$

$$\therefore \frac{dy}{dx} = \frac{1}{1+x} - \frac{(2+x)(2) - 2x(1)}{(2+x)^2} = \frac{1}{1+x} - \frac{4}{(2+x)^2} = \frac{x^2}{(2+x)^2}$$

$$\text{Now, } \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{x^2}{(2+x)^2} = 0$$

$$\Rightarrow x^2 = 0 \quad [(2+x) \neq 0 \text{ as } x > -1]$$

$$\Rightarrow x = 0$$

Since $x > -1$, point $x = 0$ divides the domain $(-1, \infty)$ in two disjoint intervals i.e., $-1 < x < 0$ and $x > 0$.

When $-1 < x < 0$, we have:

$$x < 0 \Rightarrow x^2 > 0$$

$$x > -1 \Rightarrow (2+x) > 0 \Rightarrow (2+x)^2 > 0$$

$$\therefore y' = \frac{x^2}{(2+x)^2} > 0$$

Also, when $x > 0$:

$$x > 0 \Rightarrow x^2 > 0, (2+x)^2 > 0$$

$$\therefore y' = \frac{x^2}{(2+x)^2} > 0$$

Hence, function f is increasing throughout this domain.

Ans 3.

The equation of the line is $5y - 15x = 13$.

$$5y - 15x = 13 \Rightarrow y = 3x + \frac{13}{5}$$

This is of the form $y = mx + c$.

\therefore Slope of the line = 3

If a tangent is perpendicular to the line $5y - 15x = 13$, then the slope of the tangent is $\frac{-1}{\text{slope of the line}} = \frac{-1}{3}$.

$$\Rightarrow 2x - 2 = \frac{-1}{3}$$

$$\Rightarrow 2x = \frac{-1}{3} + 2$$

$$\Rightarrow 2x = \frac{5}{3}$$

$$\Rightarrow x = \frac{5}{6}$$

$$\text{Now, } x = \frac{5}{6}$$

$$\Rightarrow y = \frac{25}{36} - \frac{10}{6} + 7 = \frac{25 - 60 + 252}{36} = \frac{217}{36}$$

Thus, the equation of the tangent passing through $\left(\frac{5}{6}, \frac{217}{36}\right)$ is given by,

$$y - \frac{217}{36} = -\frac{1}{3}\left(x - \frac{5}{6}\right)$$

$$\Rightarrow \frac{36y - 217}{36} = \frac{-1}{18}(6x - 5)$$

$$\Rightarrow 36y - 217 = -2(6x - 5)$$

$$\Rightarrow 36y - 217 = -12x + 10$$

$$\Rightarrow 36y + 12x - 227 = 0$$

Hence, the equation of the tangent line to the given curve (which is perpendicular to line $5y - 15x = 13$) is $36y + 12x - 227 = 0$.

Ans 4. Do yourself.