SOLUTION KEY CLASS XII A TEST (EX. 6.1 – 6.4) HELD ON 09-AUGUST-2011

Ans 1. Do yourself.

Ans 2.

We have,

$$y = \frac{4\sin\theta}{(2+\cos\theta)} - \theta$$

$$\therefore \frac{dy}{dx} = \frac{(2+\cos\theta)(4\cos\theta) - 4\sin\theta(-\sin\theta)}{(2+\cos\theta)^2} - 1$$

$$= \frac{8\cos\theta + 4\cos^2\theta + 4\sin^2\theta}{(2+\cos\theta)^2} - 1$$

$$= \frac{8\cos\theta + 4}{(2+\cos\theta)^2} - 1$$

Now, $\frac{dy}{dx} = 0.$

$$\Rightarrow \frac{8\cos\theta + 4}{(2+\cos\theta)^2} = 1$$

$$\Rightarrow 8\cos\theta + 4 = 4 + \cos^2\theta + 4\cos\theta$$

$$\Rightarrow \cos^2\theta - 4\cos\theta = 0$$

$$\Rightarrow \cos\theta(\cos\theta - 4) = 0$$

$$\Rightarrow \cos\theta = 0 \text{ or } \cos\theta = 4$$

Since $\cos\theta \neq 4$, $\cos\theta = 0$.

$$\cos\theta = 0 \Rightarrow \theta = \frac{\pi}{2}$$

Now,

$$\frac{dy}{dx} = \frac{8\cos\theta + 4 - (4 + \cos^2\theta + 4\cos\theta)}{(2+\cos\theta)^2} = \frac{4\cos\theta - \cos^2\theta}{(2+\cos\theta)^2} = \frac{\cos\theta(4-\cos\theta)}{(2+\cos\theta)^2}$$

In interval $\left(0, \frac{\pi}{2}\right)$, we have $\cos\theta > 0$. Also, $4 > \cos\theta \Rightarrow 4 - \cos\theta > 0$.

$$\therefore \cos\theta(4-\cos\theta) > 0 \text{ and also } (2+\cos\theta)^2 > 0$$

$$\Rightarrow \frac{\cos\theta(4-\cos\theta)}{(2+\cos\theta)^2} > 0$$

$$\Rightarrow \frac{dy}{dx} > 0$$

Therefore, y is strictly increasing in interval $\left(0, \frac{\pi}{2}\right)$.

Also, the given function is continuous at $x = 0$ and $x = \frac{\pi}{2}$.

Hence, y is increasing in interval $\left[0, \frac{\pi}{2}\right]$.

Ans3.

The equation of the given curve is $y = \sqrt{3x-2}$.

The slope of the tangent to the given curve at any point (x, y) is given by,

$$\frac{dy}{dx} = \frac{3}{2\sqrt{3x-2}}$$

The equation of the given line is $4x - 2y + 5 = 0$.

$4x - 2y + 5 = 0 \Rightarrow y = 2x + \frac{5}{2}$ (which is of the form $y = mx + c$)

$\therefore$Slope of the line $= 2$

Now, the tangent to the given curve is parallel to the line $4x - 2y - 5 = 0$ if the slope of the tangent is equal to the slope of the line.

$$\frac{3}{2\sqrt{3x-2}} = 2$$
$$\Rightarrow \sqrt{3x-2} = \frac{3}{4}$$
$$\Rightarrow 3x-2 = \frac{9}{16}$$
$$\Rightarrow 3x = \frac{9}{16} + 2 = \frac{41}{16}$$
$$\Rightarrow x = \frac{41}{48}$$

When $x = \frac{41}{48}$, $y = \sqrt{3\left(\frac{41}{48}\right) - 2} = \sqrt{\frac{41}{16} - 2} = \sqrt{\frac{41-32}{16}} = \sqrt{\frac{9}{16}} = \frac{3}{4}$.

$\therefore$Equation of the tangent passing through the point $\left(\frac{41}{48}, \frac{3}{4}\right)$ is given by,

$$y - \frac{3}{4} = 2\left(x - \frac{41}{48}\right)$$

$$\Rightarrow \frac{4y-3}{4} = 2\left(\frac{48x-41}{48}\right)$$
$$\Rightarrow 4y - 3 = \frac{48x-41}{6}$$
$$\Rightarrow 24y - 18 = 48x - 41$$
$$\Rightarrow 48x - 24y = 23$$

Hence, the equation of the required tangent is $48x - 24y = 23$.

Ans 4. Do yourself.