CLASS XII APPLICATION OF DERIVATIVES CHAPTER 6

EX. 6.5 SOLUTIONS

1. Find the maximum and minimum values, if any, of the following functions given by

(i) $f(x) = (2x - 1)^2 + 3$ (ii) $f(x) = 9x^2 + 12x + 2$

(iii) $f(x) = -(x - 1)^2 + 10$ (iv) $g(x) = x^3 + 1$

ANS:

(i) The given function is $f(x) = (2x - 1)^2 + 3$.

It can be observed that $(2x - 1)^2 \geq 0$ for every $x \in \mathbf{R}$.

Therefore, $f(x) = (2x - 1)^2 + 3 \geq 3$ for every $x \in \mathbf{R}$.

The minimum value of f is attained when $2x - 1 = 0$.

$2x - 1 = 0 \Rightarrow x = \frac{1}{2}$

$\therefore$Minimum value of $f = f\left(\frac{1}{2}\right) = \left(2 \cdot \frac{1}{2} - 1\right)^2 + 3 = 3$

Hence, function f does not have a maximum value.

(ii) The given function is $f(x) = 9x^2 + 12x + 2 = (3x + 2)^2 - 2$.

It can be observed that $(3x + 2)^2 \geq 0$ for every $x \in \mathbf{R}$.

Therefore, $f(x) = (3x + 2)^2 - 2 \geq -2$ for every $x \in \mathbf{R}$.

The minimum value of f is attained when $3x + 2 = 0$.

$3x + 2 = 0 \Rightarrow x = \frac{-2}{3}$

$\therefore$Minimum value of $f = f\left(-\frac{2}{3}\right) = \left(3\left(\frac{-2}{3}\right) + 2\right)^2 - 2 = -2$

Hence, function f does not have a maximum value.

(iii) The given function is $f(x) = -(x-1)^2 + 10$.

It can be observed that $(x-1)^2 \geq 0$ for every $x \in \mathbf{R}$.

Therefore, $f(x) = -(x-1)^2 + 10 \leq 10$ for every $x \in \mathbf{R}$.

The maximum value of f is attained when $(x-1) = 0$.

$(x-1) = 0 \Rightarrow x = 0$

$\therefore$Maximum value of $f = f(1) = -(1-1)^2 + 10 = 10$

Hence, function f does not have a minimum value.

(iv) The given function is $g(x) = x^3 + 1$.

Hence, function g neither has a maximum value nor a minimum value.

2. Find the maximum and minimum values, if any, of the following functions given by

(i) $f(x) = |x+2| - 1$ (ii) $g(x) = -|x+1| + 3$

(iii) $h(x) = \sin(2x) + 5$ (iv) $f(x) = |\sin 4x + 3|$

(v) $h(x) = x + 1, x \in (-1, 1)$

ANS:

(i) $f(x) = |x+2| - 1$

We know that $|x+2| \geq 0$ for every $x \in \mathbf{R}$.

Therefore, $f(x) = |x+2| - 1 \geq -1$ for every $x \in \mathbf{R}$.

The minimum value of f is attained when $|x+2| = 0$.

$|x+2| = 0$
$\Rightarrow x = -2$

$\therefore$Minimum value of $f = f(-2) = = |-2+2| - 1 = -1$

Hence, function f does not have a maximum value.

(ii) $g(x) = -|x+1| + 3$

We know that $-|x+1| \leq 0$ for every $x \in \mathbf{R}$.

Therefore, $g(x) = -|x+1| + 3 \leq 3$ for every $x \in \mathbf{R}$.

The maximum value of g is attained when $|x+1| = 0$.

$|x+1| = 0$
$\Rightarrow x = -1$

$\therefore$Maximum value of $g = g(-1) = -|-1+1| + 3 = 3$

Hence, function g does not have a minimum value.

(iii) $h(x) = \sin 2x + 5$

We know that $-1 \le \sin 2x \le 1$.

$\Rightarrow -1 + 5 \le \sin 2x + 5 \le 1 + 5$

$\Rightarrow 4 \le \sin 2x + 5 \le 6$

Hence, the maximum and minimum values of h are 6 and 4 respectively.

(iv) $f(x) = |\sin 4x + 3|$

We know that $-1 \le \sin 4x \le 1$.

$\Rightarrow 2 \le \sin 4x + 3 \le 4$

$\Rightarrow 2 \le |\sin 4x + 3| \le 4$

Hence, the maximum and minimum values of f are 4 and 2 respectively.

(v) $h(x) = x + 1, x \in (-1, 1)$

Here, if a point x_0 is closest to -1, then we find $\frac{x_0}{2} + 1 < x_0 + 1$ for all $x_0 \in (-1, 1)$.

Also, if x_1 is closest to 1, then $x_1 + 1 < \frac{x_1 + 1}{2} + 1$ for all $x_1 \in (-1, 1)$.

Hence, function $h(x)$ has neither maximum nor minimum value in $(-1, 1)$.

3. Find the local maxima and local minima, if any, of the following functions. Find also the local maximum and the local minimum values, as the case may be:

(i) $f(x) = x^2$ (ii) $g(x) = x^3 - 3x$

(iii) $h(x) = \sin x + \cos x, 0 < x < \frac{\pi}{2}$

(iv) $f(x) = \sin x - \cos x, \ 0 < x < 2\pi$

(v) $f(x) = x^3 - 6x^2 + 9x + 15$ (vi) $g(x) = \frac{x}{2} + \frac{2}{x}, \quad x > 0$

(vii) $g(x) = \frac{1}{x^2 + 2}$ (viii) $f(x) = x\sqrt{1 - x}, \ x > 0$

ANS:

(i) $f(x) = x^2$

$\therefore f'(x) = 2x$

Now,

$f'(x) = 0 \Rightarrow x = 0$

Thus, $x = 0$ is the only critical point which could possibly be the point of local maxima or local minima of f.

We have $f''(0) = 2$, which is positive.

Therefore, by second derivative test, $x = 0$ is a point of local minima and local minimum value of f at $x = 0$ is $f(0) = 0$.

(ii) $g(x) = x^3 - 3x$

$\therefore g'(x) = 3x^2 - 3$

Now,

$g'(x) = 0 \Rightarrow 3x^2 = 3 \Rightarrow x = \pm 1$

$g'(x) = 6x$

$g'(1) = 6 > 0$

$g'(-1) = -6 < 0$

By second derivative test, $x = 1$ is a point of local minima and local minimum value of g at $x = 1$ is $g(1) = 1^3 - 3 = 1 - 3 = -2$. However,

$x = -1$ is a point of local maxima and local maximum value of g at

$x = -1$ is $g(1) = (-1)^3 - 3(-1) = -1 + 3 = 2$.

(iii) $h(x) = \sin x + \cos x,\ 0 < x < \frac{\pi}{2}$

$$\therefore h'(x) = \cos x - \sin x$$

$$h'(x) = 0 \Rightarrow \sin x = \cos x \Rightarrow \tan x = 1 \Rightarrow x = \frac{\pi}{4} \in \left(0,\ \frac{\pi}{2}\right)$$

$$h''(x) = -\sin x - \cos x = -(\sin x + \cos x)$$

$$h''\left(\frac{\pi}{4}\right) = -\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) = -\frac{2}{\sqrt{2}} = -\sqrt{2} < 0$$

Therefore, by second derivative test, $x = \frac{\pi}{4}$ is a point of local maxima and the local maximum value of h at $x = \frac{\pi}{4}$ is

$$h\left(\frac{\pi}{4}\right) = \sin\frac{\pi}{4} + \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \sqrt{2}.$$

(iv) $f(x) = \sin x - \cos x,\ 0 < x < 2\pi$

$$\therefore f'(x) = \cos x + \sin x$$

$$f'(x) = 0 \Rightarrow \cos x = -\sin x \Rightarrow \tan x = -1 \Rightarrow x = \frac{3\pi}{4}, \frac{7\pi}{4} \in (0, 2\pi)$$

$$f''(x) = -\sin x + \cos x$$

$$f''\left(\frac{3\pi}{4}\right) = -\sin\frac{3\pi}{4} + \cos\frac{3\pi}{4} = -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} = -\sqrt{2} > 0$$

$$f''\left(\frac{7\pi}{4}\right) = -\sin\frac{7\pi}{4} + \cos\frac{7\pi}{4} = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \sqrt{2} > 0$$

Therefore, by second derivative test, $x = \frac{3\pi}{4}$ is a point of local maxima and the local maximum value of f at $x = \frac{3\pi}{4}$ is

$f\left(\frac{3\pi}{4}\right)=\sin\frac{3\pi}{4}-\cos\frac{3\pi}{4}=\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}=\sqrt{2}$. However, $x=\frac{7\pi}{4}$ is a point of local minima and the local minimum value of f at $x=\frac{7\pi}{4}$ is $f\left(\frac{7\pi}{4}\right)=\sin\frac{7\pi}{4}-\cos\frac{7\pi}{4}=-\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}=-\sqrt{2}$.

(v) $f(x) = x^3 - 6x^2 + 9x + 15$

$\therefore f'(x)=3x^2-12x+9$

$f'(x)=0 \Rightarrow 3\left(x^2-4x+3\right)=0$

$\Rightarrow 3(x-1)(x-3)=0$

$\Rightarrow x=1,\ 3$

Now, f''

$(x)=6x-12=6(x-2)$

$f''(1)=6(1-2)=-6<0$

$f''(3)=6(3-2)=6>0$

Therefore, by second derivative test, $x = 1$ is a point of local maxima and the local maximum value of f at $x = 1$ is $f(1) = 1 - 6 + 9 + 15 = 19$. However, $x = 3$ is a point of local minima and the local minimum value of f at $x = 3$ is $f(3) = 27 - 54 + 27 + 15 = 15$.

(vi) $g(x)=\frac{x}{2}+\frac{2}{x}, x>0$

$\therefore g'(x)=\frac{1}{2}-\frac{2}{x^2}$

Now,

$g'(x)=0$ gives $\frac{2}{x^2}=\frac{1}{2}\Rightarrow x^2=4\Rightarrow x=\pm 2$

Since $x > 0$, we take $x = 2$.

Now,

$g''(x)=\frac{4}{x^3}$

$g''(2)=\frac{4}{2^3}=\frac{1}{2}>0$

Therefore, by second derivative test, $x = 2$ is a point of local minima and the local minimum value of g at $x = 2$ is $g(2)=\frac{2}{2}+\frac{2}{2}=1+1=2$.

(vii) $g(x)=\frac{1}{x^2+2}$

$\therefore g'(x)=\frac{-(2x)}{\left(x^2+2\right)^2}$

$g'(x)=0\Rightarrow\frac{-2x}{\left(x^2+2\right)^2}=0\Rightarrow x=0$

Now, for values close to $x = 0$ and to the left of $0, g'(x)>0$. Also, for values close to $x = 0$ and to the right of $0, g'(x)<0$.

Therefore, by first derivative test, $x = 0$ is a point of local maxima and the local maximum value of $g(0)$ is $\frac{1}{0+2}=\frac{1}{2}$.

(viii) $f(x) = x\sqrt{1-x},\ x > 0$

$$\therefore f'(x) = \sqrt{1-x} + x \cdot \frac{1}{2\sqrt{1-x}}(-1) = \sqrt{1-x} - \frac{x}{2\sqrt{1-x}}$$

$$= \frac{2(1-x) - x}{2\sqrt{1-x}} = \frac{2-3x}{2\sqrt{1-x}}$$

$$f'(x) = 0 \Rightarrow \frac{2-3x}{2\sqrt{1-x}} = 0 \Rightarrow 2-3x = 0 \Rightarrow x = \frac{2}{3}$$

$$f''(x) = \frac{1}{2}\left[\frac{\sqrt{1-x}(-3) - (2-3x)\left(\frac{-1}{2\sqrt{1-x}}\right)}{1-x}\right]$$

$$= \frac{\sqrt{1-x}(-3) + (2-3x)\left(\frac{1}{2\sqrt{1-x}}\right)}{2(1-x)}$$

$$= \frac{-6(1-x) + (2-3x)}{4(1-x)^{\frac{3}{2}}}$$

$$= \frac{3x-4}{4(1-x)^{\frac{3}{2}}}$$

$$f''\left(\frac{2}{3}\right) = \frac{3\left(\frac{2}{3}\right) - 4}{4\left(1 - \frac{2}{3}\right)^{\frac{3}{2}}} = \frac{2-4}{4\left(\frac{1}{3}\right)^{\frac{3}{2}}} = \frac{-1}{2\left(\frac{1}{3}\right)^{\frac{3}{2}}} < 0$$

Therefore, by second derivative test, $x = \frac{2}{3}$ is a point of local maxima and the local maximum value of f at $x = \frac{2}{3}$ is

$$f\left(\frac{2}{3}\right) = \frac{2}{3}\sqrt{1 - \frac{2}{3}} = \frac{2}{3}\sqrt{\frac{1}{3}} = \frac{2}{3\sqrt{3}} = \frac{2\sqrt{3}}{9}.$$

4. Prove that the following functions do not have maxima or minima:

(i) $f(x) = e^x$ (ii) $g(x) = \log x$

(iii) $h(x) = x^3 + x^2 + x + 1$

ANS:

i. We have,

$f(x) = e^x$

$\therefore f'(x) = e^x$

Now, if $f'(x) = 0$, then $e^x = 0$. But, the exponential function can never assume 0 for any value of x.

Therefore, there does not exist $c \in \mathbf{R}$ such that $f'(c) = 0$.

Hence, function f does not have maxima or minima.

ii. We have,

$g(x) = \log x$

$\therefore g'(x) = \frac{1}{x}$

Since $\log x$ is defined for a positive number x, $g'(x) > 0$ for any x.

Therefore, there does not exist $c \in \mathbf{R}$ such that $g'(c) = 0$.

Hence, function g does not have maxima or minima.

iii. We have,

$h(x) = x^3 + x^2 + x + 1$

$\therefore h'(x) = 3x^2 + 2x + 1$

Now,

$h(x) = 0 \Rightarrow 3x^2 + 2x + 1 = 0 \Rightarrow x = \frac{-2 \pm 2\sqrt{2}\,i}{6} = \frac{-1 \pm \sqrt{2}\,i}{3} \notin \mathbf{R}$

Therefore, there does not exist $c \in \mathbf{R}$ such that $h'(c) = 0$.

Hence, function h does not have maxima or minima.

5. Find the absolute maximum value and the absolute minimum value of the following functions in the given intervals:

(i) $f(x) = x^3, x \in [-2, 2]$ (ii) $f(x) = \sin x + \cos x, x \in [0, \pi]$

(iii) $f(x) = 4x - \frac{1}{2}x^2, x \in \left[-2, \frac{9}{2}\right]$ (iv) $f(x) = (x-1)^2 + 3, x \in [-3, 1]$

ANS:

(i) The given function is $f(x) = x^3$.

$\therefore f'(x) = 3x^2$

Now,

$f'(x) = 0 \Rightarrow x = 0$

Then, we evaluate the value of f at critical point $x = 0$ and at end points of the interval $[-2, 2]$.

$f(0) = 0$

$f(-2) = (-2)^3 = -8$

$f(2) = (2)^3 = 8$

Hence, we can conclude that the absolute maximum value of f on $[-2, 2]$ is 8 occurring at $x = 2$. Also, the absolute minimum value of f on $[-2, 2]$ is -8 occurring at $x = -2$.

(ii) The given function is $f(x) = \sin x + \cos x$.

$\therefore f'(x) = \cos x - \sin x$

Now,

$f'(x) = 0 \Rightarrow \sin x = \cos x \Rightarrow \tan x = 1 \Rightarrow x = \frac{\pi}{4}$

Then, we evaluate the value of f at critical point $x = \frac{\pi}{4}$ and at the end points of the interval $[0, \pi]$.

$$f\left(\frac{\pi}{4}\right)=\sin\frac{\pi}{4}+\cos\frac{\pi}{4}=\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}=\frac{2}{\sqrt{2}}=\sqrt{2}$$
$$f(0)=\sin 0+\cos 0=0+1=1$$
$$f(\pi)=\sin\pi+\cos\pi=0-1=-1$$

Hence, we can conclude that the absolute maximum value of f on $[0, \pi]$ is $\sqrt{2}$ occurring at $x=\frac{\pi}{4}$ and the absolute minimum value of f on $[0, \pi]$ is -1 occurring at $x=\pi$.

(iii) The given function is $f(x)=4x-\frac{1}{2}x^2$.

$$\therefore f'(x)=4-\frac{1}{2}(2x)=4-x$$

Now,

$$f'(x)=0 \Rightarrow x=4$$

Then, we evaluate the value of f at critical point $x=4$ and at the end points of the interval $\left[-2, \frac{9}{2}\right]$.

$$f(4)=16-\frac{1}{2}(16)=16-8=8$$
$$f(-2)=-8-\frac{1}{2}(4)=-8-2=-10$$
$$f\left(\frac{9}{2}\right)=4\left(\frac{9}{2}\right)-\frac{1}{2}\left(\frac{9}{2}\right)^2=18-\frac{81}{8}=18-10.125=7.875$$

Hence, we can conclude that the absolute maximum value of f on $\left[-2, \frac{9}{2}\right]$ is 8 occurring at $x=4$ and the absolute minimum value of f on $\left[-2, \frac{9}{2}\right]$ is -10 occurring at $x=-2$.

(iv) The given function is $f(x)=(x-1)^2+3$.

$$\therefore f'(x)=2(x-1)$$

Now,

$$f'(x)=0 \Rightarrow 2(x-1)=0 \Rightarrow x=1$$

Then, we evaluate the value of f at critical point $x=1$ and at the end points of the interval $[-3, 1]$.

$$f(1)=(1-1)^2+3=0+3=3$$
$$f(-3)=(-3-1)^2+3=16+3=19$$

Hence, we can conclude that the absolute maximum value of f on $[-3, 1]$ is 19 occurring at $x=-3$ and the minimum value of f on $[-3, 1]$ is 3 occurring at $x=1$.

6. **Find the maximum profit that a company can make, if the profit function is given by**

$$p(x) = 41 - 24x - 18x^2$$

ANS:

The profit function is given as $p(x) = 41 - 24x - 18x^2$.

$$\therefore p'(x) = -24 - 36x$$
$$p''(x) = -36$$

Now,

$$p'(x) = 0 \Rightarrow x = \frac{-24}{36} = -\frac{2}{3}$$

Also,

$$p''\left(\frac{-2}{3}\right) = -36 < 0$$

By second derivative test, $x = -\frac{2}{3}$ is the point of local maxima of p.

$$\therefore \text{Maximum profit} = p\left(-\frac{2}{3}\right)$$
$$= 41 - 24\left(-\frac{2}{3}\right) - 18\left(-\frac{2}{3}\right)^2$$
$$= 41 + 16 - 8$$
$$= 49$$

Hence, the maximum profit that the company can make is 49 units.

7. **Find both the maximum value and the minimum value of $3x^4 - 8x^3 + 12x^2 - 48x + 25$ on the interval [0, 3].**

ANS:

Let $f(x) = 3x^4 - 8x^3 + 12x^2 - 48x + 25$.

$$\therefore f'(x) = 12x^3 - 24x^2 + 24x - 48$$
$$= 12\left(x^3 - 2x^2 + 2x - 4\right)$$
$$= 12\left[x^2(x-2) + 2(x-2)\right]$$
$$= 12(x-2)\left(x^2+2\right)$$

Now, $f'(x) = 0$ gives $x = 2$ or $x^2 + 2 = 0$ for which there are no real roots.

Therefore, we consider only $x = 2 \in [0, 3]$.

Now, we evaluate the value of f at critical point $x = 2$ and at the end points of the interval [0, 3].

$$f(2) = 3(16) - 8(8) + 12(4) - 48(2) + 25$$
$$= 48 - 64 + 48 - 96 + 25$$
$$= -39$$
$$f(0) = 3(0) - 8(0) + 12(0) - 48(0) + 25$$
$$= 25$$
$$f(3) = 3(81) - 8(27) + 12(9) - 48(3) + 25$$
$$= 243 - 216 + 108 - 144 + 25 = 16$$

Hence, we can conclude that the absolute maximum value of f on [0, 3] is 25 occurring at $x = 0$ and the absolute minimum value of f at [0, 3] is -39 occurring at $x = 2$.

8. At what points in the interval $[0, 2\pi]$, does the function $\sin 2x$ attain its maximum value?

ANS:

Let $f(x) = \sin 2x$.

$\therefore f'(x) = 2\cos 2x$

Now,

$f'(x) = 0 \Rightarrow \cos 2x = 0$

$\Rightarrow 2x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$

$\Rightarrow x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

Then, we evaluate the values of f at critical points $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$ and at the end points of the interval $[0, 2\pi]$.

$f\left(\frac{\pi}{4}\right) = \sin\frac{\pi}{2} = 1, f\left(\frac{3\pi}{4}\right) = \sin\frac{3\pi}{2} = -1$

$f\left(\frac{5\pi}{4}\right) = \sin\frac{5\pi}{2} = 1, f\left(\frac{7\pi}{4}\right) = \sin\frac{7\pi}{2} = -1$

$f(0) = \sin 0 = 0, f(2\pi) = \sin 2\pi = 0$

Hence, we can conclude that the absolute maximum value of f on $[0, 2\pi]$ is occurring at $x = \frac{\pi}{4}$ and $x = \frac{5\pi}{4}$.

9. What is the maximum value of the function $\sin x + \cos x$?

ANS:

Let $f(x) = \sin x + \cos x$.

$\therefore f'(x) = \cos x - \sin x$

$f'(x) = 0 \Rightarrow \sin x = \cos x \Rightarrow \tan x = 1 \Rightarrow x = \frac{\pi}{4}, \frac{5\pi}{4}...,$

$f''(x) = -\sin x - \cos x = -(\sin x + \cos x)$

Now, $f''(x)$ will be negative when $(\sin x + \cos x)$ is positive i.e., when $\sin x$ and $\cos x$ are both positive. Also, we know that $\sin x$ and $\cos x$ both are positive in the first quadrant. Then, $f''(x)$ will be negative when $x \in \left(0, \frac{\pi}{2}\right)$.

Thus, we consider $x = \frac{\pi}{4}$.

$f''\left(\frac{\pi}{4}\right) = -\left(\sin\frac{\pi}{4} + \cos\frac{\pi}{4}\right) = -\left(\frac{2}{\sqrt{2}}\right) = -\sqrt{2} < 0$

$\therefore$By second derivative test, f will be the maximum at $x = \frac{\pi}{4}$ and the maximum value of f is $f\left(\frac{\pi}{4}\right) = \sin\frac{\pi}{4} + \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$.

10. Find the maximum value of $2x^3 - 24x + 107$ in the interval [1, 3]. Find the maximum value of the same function in [–3, –1].

ANS:

Let $f(x) = 2x^3 - 24x + 107$.

$\therefore f'(x) = 6x^2 - 24 = 6(x^2 - 4)$

Now,

$f'(x) = 0 \Rightarrow 6(x^2 - 4) = 0 \Rightarrow x^2 = 4 \Rightarrow x = \pm 2$

We first consider the interval [1, 3].

Then, we evaluate the value of f at the critical point $x = 2 \in [1, 3]$ and at the end points of the interval [1, 3].

$f(2) = 2(8) - 24(2) + 107 = 16 - 48 + 107 = 75$

$f(1) = 2(1) - 24(1) + 107 = 2 - 24 + 107 = 85$

$f(3) = 2(27) - 24(3) + 107 = 54 - 72 + 107 = 89$

Hence, the absolute maximum value of $f(x)$ in the interval [1, 3] is 89 occurring at $x = 3$.

Next, we consider the interval $[-3, -1]$.

Evaluate the value of f at the critical point $x = -2 \in [-3, -1]$ and at the end points of the interval [1, 3].

$f(-3) = 2(-27) - 24(-3) + 107 = -54 + 72 + 107 = 125$

$f(-1) = 2(-1) - 24(-1) + 107 = -2 + 24 + 107 = 129$

$f(-2) = 2(-8) - 24(-2) + 107 = -16 + 48 + 107 = 139$

Hence, the absolute maximum value of $f(x)$ in the interval $[-3, -1]$ is 139 occurring at $x = -2$.

11. It is given that at $x = 1$, the function $x^4 - 62x^2 + ax + 9$ attains its maximum value, on the interval [0, 2]. Find the value of a.

ANS:

Let $f(x) = x^4 - 62x^2 + ax + 9$.

$\therefore f'(x) = 4x^3 - 124x + a$

It is given that function f attains its maximum value on the interval [0, 2] at $x = 1$.

$\therefore f'(1) = 0$

$\Rightarrow 4 - 124 + a = 0$

$\Rightarrow a = 120$

Hence, the value of a is 120.

12. Find the maximum and minimum values of $x + \sin 2x$ on $[0, 2\pi]$.

ANS:

Let $f(x) = x + \sin 2x$.

$\therefore f'(x) = 1 + 2\cos 2x$

Now, $f'(x) = 0 \Rightarrow \cos 2x = -\frac{1}{2} = -\cos\frac{\pi}{3} = \cos\left(\pi - \frac{\pi}{3}\right) = \cos\frac{2\pi}{3}$

$2x = 2n\pi \pm \frac{2\pi}{3}, \quad n \in \mathbf{Z}$

$\Rightarrow x = n\pi \pm \frac{\pi}{3}, \; n \in \mathbf{Z}$

$\Rightarrow x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3} \in [0, 2\pi]$

Then, we evaluate the value of f at critical points $x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$ and at the end points of the interval $[0, 2\pi]$.

$f\left(\frac{\pi}{3}\right) = \frac{\pi}{3} + \sin\frac{2\pi}{3} = \frac{\pi}{3} + \frac{\sqrt{3}}{2}$

$f\left(\frac{2\pi}{3}\right) = \frac{2\pi}{3} + \sin\frac{4\pi}{3} = \frac{2\pi}{3} - \frac{\sqrt{3}}{2}$

$f\left(\frac{4\pi}{3}\right) = \frac{4\pi}{3} + \sin\frac{8\pi}{3} = \frac{4\pi}{3} + \frac{\sqrt{3}}{2}$

$f\left(\frac{5\pi}{3}\right) = \frac{5\pi}{3} + \sin\frac{10\pi}{3} = \frac{5\pi}{3} - \frac{\sqrt{3}}{2}$

$f(0) = 0 + \sin 0 = 0$

$f(2\pi) = 2\pi + \sin 4\pi = 2\pi + 0 = 2\pi$

Hence, we can conclude that the absolute maximum value of $f(x)$ in the interval $[0, 2\pi]$ is 2π occurring at $x = 2\pi$ and the absolute minimum value of $f(x)$ in the interval $[0, 2\pi]$ is 0 occurring at $x = 0$.