CLASS XII APPLICATION OF DERIVATIVES CHAPTER 6

EX. 6.4 SOLUTIONS

1. Using differentials, find the approximate value of each of the following up to 3 places of decimal.

(i) $\sqrt{25.3}$ (ii) $\sqrt{49.5}$ (iii) $\sqrt{0.6}$

(iv) $(0.009)^{\frac{1}{3}}$ (v) $(0.999)^{\frac{1}{10}}$ (vi) $(15)^{\frac{1}{4}}$

(vii) $(26)^{\frac{1}{3}}$ (viii) $(255)^{\frac{1}{4}}$ (ix) $(82)^{\frac{1}{4}}$

(x) $(401)^{\frac{1}{2}}$ (xi) $(0.0037)^{\frac{1}{2}}$ (xii) $(26.57)^{\frac{1}{3}}$

(xiii) $(81.5)^{\frac{1}{4}}$ (xiv) $(3.968)^{\frac{3}{2}}$ (xv) $(32.15)^{\frac{1}{5}}$

ANS:

(i) $\sqrt{25.3}$

Consider $y=\sqrt{x}$. Let $x = 25$ and $\Delta x = 0.3$.

Then,

$$\Delta y=\sqrt{x+\Delta x}-\sqrt{x}=\sqrt{25.3}-\sqrt{25}=\sqrt{25.3}-5$$
$$\Rightarrow \sqrt{25.3}=\Delta y+5$$

Now, dy is approximately equal to Δy and is given by,

$$dy=\left(\frac{dy}{dx}\right)\Delta x=\frac{1}{2\sqrt{x}}(0.3) \qquad \left[\text{as } y=\sqrt{x}\right]$$

$$=\frac{1}{2\sqrt{25}}(0.3)=0.03$$

Hence, the approximate value of $\sqrt{25.3}$ is 0.03 + 5 = 5.03

(ii) $\sqrt{49.5}$

Consider $y=\sqrt{x}$. Let $x = 49$ and $\Delta x = 0.5$.

Then,

$$\Delta y=\sqrt{x+\Delta x}-\sqrt{x}=\sqrt{49.5}-\sqrt{49}=\sqrt{49.5}-7$$
$$\Rightarrow \sqrt{49.5}=7+\Delta y$$

Now, dy is approximately equal to Δy and is given by,

$$dy=\left(\frac{dy}{dx}\right)\Delta x=\frac{1}{2\sqrt{x}}(0.5) \qquad \left[\text{as } y=\sqrt{x}\right]$$
$$=\frac{1}{2\sqrt{49}}(0.5)=\frac{1}{14}(0.5)=0.035$$

Hence, the approximate value of $\sqrt{49.5}$ is 7 + 0.035 = 7.035.

(iii) $\sqrt{0.6}$

Consider $y=\sqrt{x}$. Let $x = 1$ and $\Delta x = -0.4$.

Then,

$$\Delta y=\sqrt{x+\Delta x}-\sqrt{x}=\sqrt{0.6}-1$$
$$\Rightarrow \sqrt{0.6}=1+\Delta y$$

Now, dy is approximately equal to Δy and is given by,

$$dy = \left(\frac{dy}{dx}\right)\Delta x = \frac{1}{2\sqrt{x}}(\Delta x) \qquad \left[\text{as } y = \sqrt{x}\right]$$

$$= \frac{1}{2}(-0.4) = -0.2$$

Hence, the approximate value of $\sqrt{0.6}$ is $1 + (-0.2) = 1 - 0.2 = 0.8$.

(iv) $(0.009)^{\frac{1}{3}}$

Consider $y = x^{\frac{1}{3}}$. Let $x = 0.008$ and $\Delta x = 0.001$.

Then,

$$\Delta y = (x + \Delta x)^{\frac{1}{3}} - (x)^{\frac{1}{3}} = (0.009)^{\frac{1}{3}} - (0.008)^{\frac{1}{3}} = (0.009)^{\frac{1}{3}} - 0.2$$

$$\Rightarrow (0.009)^{\frac{1}{3}} = 0.2 + \Delta y$$

Now, dy is approximately equal to Δy and is given by,

$$dy = \left(\frac{dy}{dx}\right)\Delta x = \frac{1}{3(x)^{\frac{2}{3}}}(\Delta x) \qquad \left[\text{as } y = x^{\frac{1}{3}}\right]$$

$$= \frac{1}{3 \times 0.04}(0.001) = \frac{0.001}{0.12} = 0.008$$

Hence, the approximate value of $(0.009)^{\frac{1}{3}}$ is $0.2 + 0.008 = 0.208$.

(v) $(0.999)^{\frac{1}{10}}$

Consider $y = (x)^{\frac{1}{10}}$. Let $x = 1$ and $\Delta x = -0.001$.

Then,

$$\Delta y = (x + \Delta x)^{\frac{1}{10}} - (x)^{\frac{1}{10}} = (0.999)^{\frac{1}{10}} - 1$$

$$\Rightarrow (0.999)^{\frac{1}{10}} = 1 + \Delta y$$

Now, dy is approximately equal to Δy and is given by,

$$dy = \left(\frac{dy}{dx}\right)\Delta x = \frac{1}{10(x)^{\frac{9}{10}}}(\Delta x) \qquad \left[\text{as } y = (x)^{\frac{1}{10}}\right]$$

$$= \frac{1}{10}(-0.001) = -0.0001$$

Hence, the approximate value of $(0.999)^{\frac{1}{10}}$ is 1 + (−0.0001) = 0.9999.

(vi) $(15)^{\frac{1}{4}}$

Consider $y = x^{\frac{1}{4}}$. Let $x = 16$ and $\Delta x = -1$.

Then,

$$\Delta y = (x+\Delta x)^{\frac{1}{4}} - x^{\frac{1}{4}} = (15)^{\frac{1}{4}} - (16)^{\frac{1}{4}} = (15)^{\frac{1}{4}} - 2$$
$$\Rightarrow (15)^{\frac{1}{4}} = 2 + \Delta y$$

Now, dy is approximately equal to Δy and is given by,

$$dy = \left(\frac{dy}{dx}\right)\Delta x = \frac{1}{4(x)^{\frac{3}{4}}}(\Delta x) \qquad \left[\text{as } y = x^{\frac{1}{4}}\right]$$
$$= \frac{1}{4(16)^{\frac{3}{4}}}(-1) = \frac{-1}{4\times 8} = \frac{-1}{32} = -0.03125$$

Hence, the approximate value of $(15)^{\frac{1}{4}}$ is 2 + (−0.03125) = 1.96875.

(vii) $(26)^{\frac{1}{3}}$

Consider $y = (x)^{\frac{1}{3}}$. Let $x = 27$ and $\Delta x = -1$.

Then,

$$\Delta y = (x+\Delta x)^{\frac{1}{3}} - (x)^{\frac{1}{3}} = (26)^{\frac{1}{3}} - (27)^{\frac{1}{3}} = (26)^{\frac{1}{3}} - 3$$
$$\Rightarrow (26)^{\frac{1}{3}} = 3 + \Delta y$$

Now, dy is approximately equal to Δy and is given by,

$$dy = \left(\frac{dy}{dx}\right)\Delta x = \frac{1}{3(x)^{\frac{2}{3}}}(\Delta x) \qquad \left[\text{as } y = (x)^{\frac{1}{3}}\right]$$
$$= \frac{1}{3(27)^{\frac{2}{3}}}(-1) = \frac{-1}{27} = -0.0\overline{370}$$

Hence, the approximate value of $(26)^{\frac{1}{3}}$ is 3 + (−0.0370) = 2.9629.

(viii) $(255)^{\frac{1}{4}}$

Consider $y=(x)^{\frac{1}{4}}$. Let $x = 256$ and $\Delta x = -1$.

Then,

$$\Delta y=(x+\Delta x)^{\frac{1}{4}}-(x)^{\frac{1}{4}}=(255)^{\frac{1}{4}}-(256)^{\frac{1}{4}}=(255)^{\frac{1}{4}}-4$$
$$\Rightarrow (255)^{\frac{1}{4}}=4+\Delta y$$

Now, dy is approximately equal to Δy and is given by,

$$dy=\left(\frac{dy}{dx}\right)\Delta x=\frac{1}{4(x)^{\frac{3}{4}}}(\Delta x) \qquad \left[\text{as } y=x^{\frac{1}{4}}\right]$$
$$=\frac{1}{4(256)^{\frac{3}{4}}}(-1)=\frac{-1}{4\times 4^3}=-0.0039$$

Hence, the approximate value of $(255)^{\frac{1}{4}}$ is $4 + (-0.0039) = 3.9961$.

(ix) $(82)^{\frac{1}{4}}$

Consider $y=x^{\frac{1}{4}}$. Let $x = 81$ and $\Delta x = 1$.

Then,

$$\Delta y=(x+\Delta x)^{\frac{1}{4}}-(x)^{\frac{1}{4}}=(82)^{\frac{1}{4}}-(81)^{\frac{1}{4}}=(82)^{\frac{1}{4}}-3$$
$$\Rightarrow (82)^{\frac{1}{4}}=\Delta y+3$$

Now, dy is approximately equal to Δy and is given by,

$$dy=\left(\frac{dy}{dx}\right)\Delta x=\frac{1}{4(x)^{\frac{3}{4}}}(\Delta x) \qquad \left[\text{as } y=x^{\frac{1}{4}}\right]$$
$$=\frac{1}{4(81)^{\frac{3}{4}}}(1)=\frac{1}{4(3)^3}=\frac{1}{108}=0.009$$

Hence, the approximate value of $(82)^{\frac{1}{4}}$ is $3 + 0.009 = 3.009$.

(x) $(401)^{\frac{1}{2}}$

Consider $y = x^{\frac{1}{2}}$. Let $x = 400$ and $\Delta x = 1$.

Then,

$$\Delta y = \sqrt{x+\Delta x} - \sqrt{x} = \sqrt{401} - \sqrt{400} = \sqrt{401} - 20$$
$$\Rightarrow \sqrt{401} = 20 + \Delta y$$

Now, dy is approximately equal to Δy and is given by,

$$dy = \left(\frac{dy}{dx}\right)\Delta x = \frac{1}{2\sqrt{x}}(\Delta x) \qquad \left[\text{as } y = x^{\frac{1}{2}}\right]$$
$$= \frac{1}{2\times 20}(1) = \frac{1}{40} = 0.025$$

Hence, the approximate value of $\sqrt{401}$ is 20 + 0.025 = 20.025.

(xi) $(0.0037)^{\frac{1}{2}}$

Consider $y = x^{\frac{1}{2}}$. Let $x = 0.0036$ and $\Delta x = 0.0001$.

Then,

$$\Delta y = (x+\Delta x)^{\frac{1}{2}} - (x)^{\frac{1}{2}} = (0.0037)^{\frac{1}{2}} - (0.0036)^{\frac{1}{2}} = (0.0037)^{\frac{1}{2}} - 0.06$$
$$\Rightarrow (0.0037)^{\frac{1}{2}} = 0.06 + \Delta y$$

Now, dy is approximately equal to Δy and is given by,

$$dy = \left(\frac{dy}{dx}\right)\Delta x = \frac{1}{2\sqrt{x}}(\Delta x) \qquad \left[\text{as } y = x^{\frac{1}{2}}\right]$$
$$= \frac{1}{2\times 0.06}(0.0001)$$
$$= \frac{0.0001}{0.12} = 0.00083$$

Thus, the approximate value of $(0.0037)^{\frac{1}{2}}$ is 0.06 + 0.00083 = 0.06083.

(xii) $(26.57)^{\frac{1}{3}}$

Consider $y = x^{\frac{1}{3}}$. Let $x = 27$ and $\Delta x = -0.43$.

Then,

$$\Delta y = (x+\Delta x)^{\frac{1}{3}} - x^{\frac{1}{3}} = (26.57)^{\frac{1}{3}} - (27)^{\frac{1}{3}} = (26.57)^{\frac{1}{3}} - 3$$
$$\Rightarrow (26.57)^{\frac{1}{3}} = 3 + \Delta y$$

Now, dy is approximately equal to Δy and is given by,

$$dy = \left(\frac{dy}{dx}\right)\Delta x = \frac{1}{3(x)^{\frac{2}{3}}}(\Delta x) \qquad \left[\text{as } y = x^{\frac{1}{3}}\right]$$
$$= \frac{1}{3(9)}(-0.43)$$
$$= \frac{-0.43}{27} = -0.015$$

Hence, the approximate value of $(26.57)^{\frac{1}{3}}$ is 3 + (−0.015) = 2.984.

(xiii) $(81.5)^{\frac{1}{4}}$

Consider $y = x^{\frac{1}{4}}$. Let $x = 81$ and $\Delta x = 0.5$.

Then,

$$\Delta y = (x+\Delta x)^{\frac{1}{4}} - (x)^{\frac{1}{4}} = (81.5)^{\frac{1}{4}} - (81)^{\frac{1}{4}} = (81.5)^{\frac{1}{4}} - 3$$
$$\Rightarrow (81.5)^{\frac{1}{4}} = 3 + \Delta y$$

Now, dy is approximately equal to Δy and is given by,

$$dy = \left(\frac{dy}{dx}\right)\Delta x = \frac{1}{4(x)^{\frac{3}{4}}}(\Delta x) \qquad \left[\text{as } y = x^{\frac{1}{4}}\right]$$
$$= \frac{1}{4(3)^3}(0.5) = \frac{0.5}{108} = 0.0046$$

Hence, the approximate value of $(81.5)^{\frac{1}{4}}$ is 3 + 0.0046 = 3.0046.

(xiv) $(3.968)^{\frac{3}{2}}$

Consider $y = x^{\frac{3}{2}}$. Let $x = 4$ and $\Delta x = -0.032$.

Then,

$$\Delta y = (x+\Delta x)^{\frac{3}{2}} - x^{\frac{3}{2}} = (3.968)^{\frac{3}{2}} - (4)^{\frac{3}{2}} = (3.968)^{\frac{3}{2}} - 8$$
$$\Rightarrow (3.968)^{\frac{3}{2}} = 8 + \Delta y$$

Now, dy is approximately equal to Δy and is given by,

$$dy = \left(\frac{dy}{dx}\right)\Delta x = \frac{3}{2}(x)^{\frac{1}{2}}(\Delta x) \qquad \left[\text{as } y = x^{\frac{3}{2}}\right]$$
$$= \frac{3}{2}(2)(-0.032)$$
$$= -0.096$$

Hence, the approximate value of $(3.968)^{\frac{3}{2}}$ is $8 + (-0.096) = 7.904$.

(xv) $(32.15)^{\frac{1}{5}}$

Consider $y = x^{\frac{1}{5}}$. Let $x = 32$ and $\Delta x = 0.15$.

Then,

$$\Delta y = (x+\Delta x)^{\frac{1}{5}} - x^{\frac{1}{5}} = (32.15)^{\frac{1}{5}} - (32)^{\frac{1}{5}} = (32.15)^{\frac{1}{5}} - 2$$
$$\Rightarrow (32.15)^{\frac{1}{5}} = 2 + \Delta y$$

Now, dy is approximately equal to Δy and is given by,

$$dy = \left(\frac{dy}{dx}\right)\Delta x = \frac{1}{5(x)^{\frac{4}{5}}}\cdot(\Delta x) \qquad \left[\text{as } y = x^{\frac{1}{5}}\right]$$
$$= \frac{1}{5\times(2)^4}(0.15)$$
$$= \frac{0.15}{80} = 0.00187$$

Hence, the approximate value of $(32.15)^{\frac{1}{5}}$ is $2 + 0.00187 = 2.00187$.

2. Find the approximate value of $f(2.01)$, where $f(x) = 4x^2 + 5x + 2$.

ANS:

Let $x = 2$ and $\Delta x = 0.01$. Then, we have:

$f(2.01) = f(x + \Delta x) = 4(x + \Delta x)^2 + 5(x + \Delta x) + 2$

Now, $\Delta y = f(x + \Delta x) - f(x)$

$\therefore f(x + \Delta x) = f(x) + \Delta y$

$\approx f(x) + f'(x) \cdot \Delta x \qquad (\text{as } dx = \Delta x)$

$$\begin{aligned}\Rightarrow f(2.01) &\approx (4x^2 + 5x + 2) + (8x + 5)\Delta x \\ &= \left[4(2)^2 + 5(2) + 2\right] + \left[8(2) + 5\right](0.01) \qquad [\text{as } x = 2,\ \Delta x = 0.01] \\ &= (16 + 10 + 2) + (16 + 5)(0.01) \\ &= 28 + (21)(0.01) \\ &= 28 + 0.21 \\ &= 28.21\end{aligned}$$

Hence, the approximate value of $f(2.01)$ is 28.21.

3. Find the approximate value of $f(5.001)$, where $f(x) = x^3 - 7x^2 + 15$.

ANS:

Let $x = 5$ and $\Delta x = 0.001$. Then, we have:

$f(5.001) = f(x + \Delta x) = (x + \Delta x)^3 - 7(x + \Delta x)^2 + 15$

Now, $\Delta y = f(x + \Delta x) - f(x)$

$\therefore f(x + \Delta x) = f(x) + \Delta y$

$\approx f(x) + f'(x) \cdot \Delta x \qquad (\text{as } dx = \Delta x)$

$$\begin{aligned}\Rightarrow f(5.001) &\approx (x^3 - 7x^2 + 15) + (3x^2 - 14x)\Delta x \\ &= \left[(5)^3 - 7(5)^2 + 15\right] + \left[3(5)^2 - 14(5)\right](0.001) \qquad [x = 5, \Delta x = 0.001] \\ &= (125 - 175 + 15) + (75 - 70)(0.001) \\ &= -35 + (5)(0.001) \\ &= -35 + 0.005 \\ &= -34.995\end{aligned}$$

Hence, the approximate value of $f(5.001)$ is -34.995.

4. Find the approximate change in the volume V of a cube of side x metres caused by increasing the side by 1%.

ANS:

The volume of a cube (V) of side x is given by $V = x^3$.

$$\therefore dV = \left(\frac{dV}{dx}\right)\Delta x$$
$$= \left(3x^2\right)\Delta x$$
$$= \left(3x^2\right)\left(0.01x\right) \qquad \left[\text{as } 1\% \text{ of } x \text{ is } 0.01x\right]$$
$$= 0.03x^3$$

Hence, the approximate change in the volume of the cube is $0.03x^3$ m^3.

5. Find the approximate change in the surface area of a cube of side x metres caused by decreasing the side by 1%.

ANS:

The surface area of a cube (S) of side x is given by $S = 6x^2$.

$$\therefore \frac{dS}{dx} = \left(\frac{dS}{dx}\right)\Delta x$$
$$= \left(12x\right)\Delta x$$
$$= \left(12x\right)\left(0.01x\right) \qquad \left[\text{as } 1\% \text{ of } x \text{ is } 0.01x\right]$$
$$= 0.12x^2$$

Hence, the approximate change in the surface area of the cube is $0.12x^2$ m^2.

6. If the radius of a sphere is measured as 7 m with an error of 0.02 m, then find the approximate error in calculating its volume.

ANS:

Let r be the radius of the sphere and Δr be the error in measuring the radius.

Then,

$r = 7$ m and $\Delta r = 0.02$ m

Now, the volume V of the sphere is given by,

$$V = \frac{4}{3}\pi r^3$$
$$\therefore \frac{dV}{dr} = 4\pi r^2$$
$$\therefore dV = \left(\frac{dV}{dr}\right)\Delta r$$
$$= \left(4\pi r^2\right)\Delta r$$
$$= 4\pi(7)^2(0.02)\ \text{m}^3 = 3.92\pi\ \text{m}^3$$

Hence, the approximate error in calculating the volume is $3.92\pi\ \text{m}^3$.

7. If the radius of a sphere is measured as 9 m with an error of 0.03 m, then find the approximate error in calculating its surface area.

ANS:

Let r be the radius of the sphere and Δr be the error in measuring the radius.

Then,

$r = 9$ m and $\Delta r = 0.03$ m

Now, the surface area of the sphere (S) is given by,

$S = 4\pi r^2$

$$\therefore \frac{dS}{dr} = 8\pi r$$
$$\therefore dS = \left(\frac{dS}{dr}\right)\Delta r$$
$$= (8\pi r)\Delta r$$
$$= 8\pi(9)(0.03)\ \text{m}^2$$
$$= 2.16\pi\ \text{m}^2$$

Hence, the approximate error in calculating the surface area is $2.16\pi\ \text{m}^2$.

8. If $f(x) = 3x^2 + 15x + 5$, then the approximate value of $f(3.02)$ is

(A) 47.66 (B) 57.66 (C) 67.66 (D) 77.66

ANS:

Let $x = 3$ and $\Delta x = 0.02$. Then, we have:

$$f(3.02) = f(x+\Delta x) = 3(x+\Delta x)^2 + 15(x+\Delta x) + 5$$

Now, $\Delta y = f(x+\Delta x) - f(x)$

$$\Rightarrow f(x+\Delta x) = f(x) + \Delta y$$

$$\approx f(x) + f'(x)\Delta x \qquad (\text{As } dx = \Delta x)$$

$$\Rightarrow f(3.02) \approx (3x^2 + 15x + 5) + (6x + 15)\Delta x$$

$$= \left[3(3)^2 + 15(3) + 5\right] + \left[6(3) + 15\right](0.02) \qquad [\text{As } x = 3,\ \Delta x = 0.02]$$

$$= (27 + 45 + 5) + (18 + 15)(0.02)$$

$$= 77 + (33)(0.02)$$

$$= 77 + 0.66$$

$$= 77.66$$

Hence, the approximate value of $f(3.02)$ is 77.66.

The correct answer is D.

9. The approximate change in the volume of a cube of side x metres caused by increasing the side by 3% is

(A) $0.06\,x^3$ m^3 (B) $0.6\,x^3$ m^3 (C) $0.09\,x^3$ m^3 (D) $0.9\,x^3$ m^3

ANS:

The volume of a cube (V) of side x is given by $V = x^3$.

$$\therefore dV = \left(\frac{dV}{dx}\right)\Delta x$$

$$= (3x^2)\Delta x$$

$$= (3x^2)(0.03x) \qquad [\text{As } 3\% \text{ of } x \text{ is } 0.03x]$$

$$= 0.09x^3 \text{ m}^3$$

Hence, the approximate change in the volume of the cube is $0.09x^3$ m^3.

The correct answer is C.