CLASS XII APPLICATION OF DERIVATIVES CHAPTER 6

EX. 6.2 SOLUTIONS

1. Show that the function given by $f(x) = 3x + 17$ is strictly increasing on **R**.

Let x_1 and x_2 be any two numbers in **R**.

Then, we have:

$$x_1 < x_2 \Rightarrow 3x_1 < 3x_2 \Rightarrow 3x_1 + 17 < 3x_2 + 17 \Rightarrow f(x_1) < f(x_2)$$

Hence, f is strictly increasing on **R**.

Alternate method:

$f'(x) = 3 > 0$, in every interval of **R**.

Thus, the function is strictly increasing on **R**.

2. Show that the function given by $f(x) = e^{2x}$ is strictly increasing on **R**.

Let x_1 and x_2 be any two numbers in **R**.

Then, we have:

$$x_1 < x_2 \Rightarrow 2x_1 < 2x_2 \Rightarrow e^{2x_1} < e^{2x_2} \Rightarrow f(x_1) < f(x_2)$$

Hence, f is strictly increasing on **R**.

3. Show that the function given by $f(x) = \sin x$ is

(a) strictly increasing in $\left(0, \frac{\pi}{2}\right)$ (b) strictly decreasing in $\left(\frac{\pi}{2}, \pi\right)$

(c) neither increasing nor decreasing in $(0, \pi)$

ANS :

The given function is $f(x) = \sin x$.

$\therefore f'(x) = \cos x$

(a) Since for each $x \in \left(0, \frac{\pi}{2}\right)$, $\cos x > 0$, we have $f'(x) > 0$.

Hence, f is strictly increasing in $\left(0, \frac{\pi}{2}\right)$.

(b) Since for each $x \in \left(\frac{\pi}{2}, \pi\right)$, $\cos x < 0$, we have $f'(x) < 0$.

Hence, f is strictly decreasing in $\left(\frac{\pi}{2}, \pi\right)$.

(c) From the results obtained in (a) and (b), it is clear that f is neither increasing nor decreasing in $(0, \pi)$.

4. Find the intervals in which the function f given by $f(x) = 2x^2 - 3x$ is

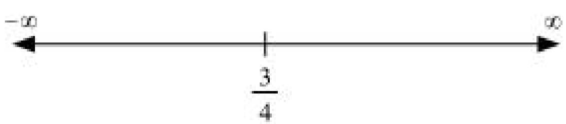
(a) strictly increasing (b) strictly decreasing

The given function is $f(x) = 2x^2 - 3x$.

$$f'(x) = 4x - 3$$

$$\therefore f'(x) = 0 \Rightarrow x = \frac{3}{4}$$

Now, the point $\frac{3}{4}$ divides the real line into two disjoint intervals i.e., $\left(-\infty, \frac{3}{4}\right)$ and $\left(\frac{3}{4}, \infty\right)$.



In interval $\left(-\infty, \frac{3}{4}\right)$, $f'(x) = 4x - 3 < 0$.

Hence, the given function (f) is strictly decreasing in interval $\left(-\infty, \frac{3}{4}\right)$.

In interval $\left(\frac{3}{4}, \infty\right)$, $f'(x) = 4x - 3 > 0$.

Hence, the given function (f) is strictly increasing in interval $\left(\frac{3}{4}, \infty\right)$.

5. Find the intervals in which the function f given by $f(x) = 2x^3 - 3x^2 - 36x + 7$ is

(a) strictly increasing (b) strictly decreasing

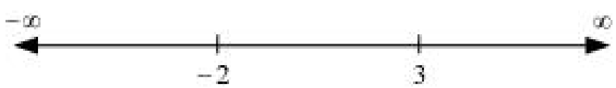
The given function is $f(x) = 2x^3 - 3x^2 - 36x + 7$.

$$f'(x) = 6x^2 - 6x - 36 = 6(x^2 - x - 6) = 6(x+2)(x-3)$$

$$\therefore f'(x) = 0 \Rightarrow x = -2, 3$$

The points $x = -2$ and $x = 3$ divide the real line into three disjoint intervals i.e.,

$(-\infty, -2), (-2, 3)$, and $(3, \infty)$.



In intervals $(-\infty, -2)$ and $(3, \infty)$, $f'(x)$ is positive while in interval

$(-2, 3)$, $f'(x)$ is negative.

Hence, the given function (f) is strictly increasing in intervals

$(-\infty, -2)$ and $(3, \infty)$, while function (f) is strictly decreasing in interval

$(-2, 3)$.

6. Find the intervals in which the following functions are strictly increasing or decreasing:

(a) $x^2 + 2x - 5$ (b) $10 - 6x - 2x^2$

(c) $-2x^3 - 9x^2 - 12x + 1$ (d) $6 - 9x - x^2$

(e) $(x + 1)^3 (x - 3)^3$

(a) We have,

$$f(x) = x^2 + 2x - 5$$

$$\therefore f'(x) = 2x + 2$$

Now,

$$f'(x) = 0 \Rightarrow x = -1$$

Point $x = -1$ divides the real line into two disjoint intervals i.e., $(-\infty, -1)$ and $(-1, \infty)$.

In interval $(-\infty, -1)$, $f'(x) = 2x + 2 < 0$.

$\therefore f$ is strictly decreasing in interval $(-\infty, -1)$.

Thus, f is strictly decreasing for $x < -1$.

In interval $(-1, \infty)$, $f'(x) = 2x + 2 > 0$.

$\therefore f$ is strictly increasing in interval $(-1, \infty)$.

Thus, f is strictly increasing for $x > -1$.

(b) We have,

$$f(x) = 10 - 6x - 2x^2$$

$$\therefore f'(x) = -6 - 4x$$

Now,

$$f'(x) = 0 \Rightarrow x = -\frac{3}{2}$$

The point $x = -\frac{3}{2}$ divides the real line into two disjoint intervals i.e., $\left(-\infty, -\frac{3}{2}\right)$ and $\left(-\frac{3}{2}, \infty\right)$.

In interval $\left(-\infty,-\frac{3}{2}\right)$ i.e., when $x<-\frac{3}{2}$, $f'(x)=-6-4x<0$.

$\therefore f$ is strictly increasing for $x<-\frac{3}{2}$.

In interval $\left(-\frac{3}{2},\infty\right)$ i.e., when $x>-\frac{3}{2}$, $f'(x)=-6-4x<0$.

$\therefore f$ is strictly decreasing for $x>-\frac{3}{2}$.

(c) We have,

$f(x) = -2x^3 - 9x^2 - 12x + 1$

$\therefore f'(x)=-6x^2-18x-12=-6\left(x^2+3x+2\right)=-6(x+1)(x+2)$

Now,

$f'(x)=0 \Rightarrow x=-1$ and $x=-2$

Points $x=-1$ and $x=-2$ divide the real line into three disjoint intervals i.e., $(-\infty,-2)$, $(-2,-1)$, and $(-1,\infty)$.

In intervals $(-\infty,-2)$ and $(-1,\infty)$ i.e., when $x<-2$ and $x>-1$,

$f'(x)=-6(x+1)(x+2)<0$.

$\therefore f$ is strictly decreasing for $x<-2$ and $x>-1$.

Now, in interval $(-2,-1)$ i.e., when $-2<x<-1$, $f'(x)=-6(x+1)(x+2)>0$.

$\therefore f$ is strictly increasing for $-2<x<-1$.

(d) We have,

$f(x) = 6 - 9x - x^2$

$\therefore f'(x) = -9 - 2x$

Now, f'

$(x) = 0$ gives $x = -\frac{9}{2}$

The point $x = -\frac{9}{2}$ divides the real line into two disjoint intervals i.e., $\left(-\infty, -\frac{9}{2}\right)$ and $\left(-\frac{9}{2}, \infty\right)$.

In interval $\left(-\infty, -\frac{9}{2}\right)$ i.e., for $x < -\frac{9}{2}$, $f'(x) = -9 - 2x > 0$.

$\therefore f$ is strictly increasing for $x < -\frac{9}{2}$.

In interval $\left(-\frac{9}{2}, \infty\right)$ i.e., for $x > -\frac{9}{2}$, $f'(x) = -9 - 2x < 0$.

$\therefore f$ is strictly decreasing for $x > -\frac{9}{2}$.

(e) We have,

$f(x) = (x + 1)^3 (x - 3)^3$

$$\begin{aligned} f'(x) &= 3(x+1)^2(x-3)^3 + 3(x-3)^2(x+1)^3 \\ &= 3(x+1)^2(x-3)^2[x-3+x+1] \\ &= 3(x+1)^2(x-3)^2(2x-2) \\ &= 6(x+1)^2(x-3)^2(x-1) \end{aligned}$$

Now,

$f'(x) = 0 \Rightarrow x = -1, 3, 1$

The points $x = -1$, $x = 1$, and $x = 3$ divide the real line into four disjoint intervals i.e., $(-\infty, -1)$, $(-1, 1)$, $(1, 3)$, and $(3, \infty)$.

In intervals $(-\infty, -1)$ and $(-1, 1)$, $f'(x) = 6(x+1)^2(x-3)^2(x-1) < 0$.

$\therefore f$ is strictly decreasing in intervals $(-\infty, -1)$ and $(-1, 1)$.

In intervals $(1, 3)$ and $(3, \infty)$, $f'(x) = 6(x+1)^2(x-3)^2(x-1) > 0$.

$\therefore f$ is strictly increasing in intervals $(1, 3)$ and $(3, \infty)$.

7. Show that $y=\log(1+x)-\dfrac{2x}{2+x}$, $x>-1$, is an increasing function of x throughout its domain.

$$y=\log(1+x)-\frac{2x}{2+x}$$

$$\therefore \frac{dy}{dx}=\frac{1}{1+x}-\frac{(2+x)(2)-2x(1)}{(2+x)^2}=\frac{1}{1+x}-\frac{4}{(2+x)^2}=\frac{x^2}{(2+x)^2}$$

Now, $\dfrac{dy}{dx}=0$

$$\Rightarrow \frac{x^2}{(2+x)^2}=0$$

$$\Rightarrow x^2=0 \qquad [(2+x)\neq 0 \text{ as } x>-1]$$

$$\Rightarrow x=0$$

Since $x>-1$, point $x=0$ divides the domain $(-1,\infty)$ in two disjoint intervals i.e., $-1<x<0$ and $x>0$.

When $-1<x<0$, we have:

$$x<0\Rightarrow x^2>0$$

$$x>-1\Rightarrow (2+x)>0\Rightarrow (2+x)^2>0$$

$$\therefore y'=\frac{x^2}{(2+x)^2}>0$$

Also, when $x>0$:

$$x>0\Rightarrow x^2>0,\ (2+x)^2>0$$

$$\therefore y'=\frac{x^2}{(2+x)^2}>0$$

Hence, function f is increasing throughout this domain.

8. Find the values of x for which $y = [x(x-2)]^2$ is an increasing function.

We have,

$$y = \left[x(x-2)\right]^2 = \left[x^2 - 2x\right]^2$$

$$\therefore \frac{dy}{dx} = y' = 2\left(x^2 - 2x\right)\left(2x-2\right) = 4x\left(x-2\right)\left(x-1\right)$$

$$\therefore \frac{dy}{dx} = 0 \Rightarrow x = 0, x = 2, x = 1.$$

The points $x = 0$, $x = 1$, and $x = 2$ divide the real line into four disjoint intervals i.e., $(-\infty, 0)$, $(0,1)$ $(1,2)$, and $(2, \infty)$.

In intervals $(-\infty, 0)$ and $(1,2)$, $\frac{dy}{dx} < 0$.

$\therefore y$ is strictly decreasing in intervals $(-\infty, 0)$ and $(1,2)$.

However, in intervals (0, 1) and (2, ∞), $\frac{dy}{dx} > 0$.

$\therefore y$ is strictly increasing in intervals (0, 1) and (2, ∞).

$\therefore y$ is strictly increasing for $0 < x < 1$ and $x > 2$.

9. Prove that $y=\frac{4\sin\theta}{(2+\cos\theta)}-\theta$ is an increasing function of θ in $\left[0,\frac{\pi}{2}\right]$.

We have,

$$y=\frac{4\sin\theta}{(2+\cos\theta)}-\theta$$

$$\therefore \frac{dy}{dx}=\frac{(2+\cos\theta)(4\cos\theta)-4\sin\theta(-\sin\theta)}{(2+\cos\theta)^2}-1$$

$$=\frac{8\cos\theta+4\cos^2\theta+4\sin^2\theta}{(2+\cos\theta)^2}-1$$

$$=\frac{8\cos\theta+4}{(2+\cos\theta)^2}-1$$

Now, $\frac{dy}{dx}=0.$

$$\Rightarrow \frac{8\cos\theta+4}{(2+\cos\theta)^2}=1$$

$$\Rightarrow 8\cos\theta+4=4+\cos^2\theta+4\cos\theta$$

$$\Rightarrow \cos^2\theta-4\cos\theta=0$$

$$\Rightarrow \cos\theta(\cos\theta-4)=0$$

$$\Rightarrow \cos\theta=0 \text{ or } \cos\theta=4$$

Since $\cos\theta\neq4$, $\cos\theta=0$.

$$\cos\theta=0\Rightarrow\theta=\frac{\pi}{2}$$

Now,

$$\frac{dy}{dx}=\frac{8\cos\theta+4-(4+\cos^2\theta+4\cos\theta)}{(2+\cos\theta)^2}=\frac{4\cos\theta-\cos^2\theta}{(2+\cos\theta)^2}=\frac{\cos\theta(4-\cos\theta)}{(2+\cos\theta)^2}$$

In interval $\left(0,\ \frac{\pi}{2}\right)$, we have $\cos\theta>0$. Also, $4>\cos\theta \Rightarrow 4-\cos\theta>0$.

$$\therefore \cos\theta(4-\cos\theta)>0 \text{ and also } (2+\cos\theta)^2>0$$

$$\Rightarrow \frac{\cos\theta(4-\cos\theta)}{(2+\cos\theta)^2}>0$$

$$\Rightarrow \frac{dy}{dx}>0$$

Therefore, y is strictly increasing in interval $\left(0,\ \frac{\pi}{2}\right)$.

Also, the given function is continuous at $x=0$ and $x=\frac{\pi}{2}$.

Hence, y is increasing in interval $\left[0,\frac{\pi}{2}\right]$.

10. Prove that the logarithmic function is strictly increasing on $(0, \infty)$.

The given function is $f(x) = \log x$.

$\therefore f'(x) = \frac{1}{x}$

It is clear that for $x > 0$, $f'(x) = \frac{1}{x} > 0$.

Hence, $f(x) = \log x$ is strictly increasing in interval $(0, \infty)$.

11. Prove that the function f given by $f(x) = x^2 - x + 1$ is neither strictly increasing nor strictly decreasing on $(-1, 1)$.

The given function is $f(x) = x^2 - x + 1$.

$\therefore f'(x) = 2x - 1$

Now, $f'(x) = 0 \Rightarrow x = \frac{1}{2}$.

The point $\frac{1}{2}$ divides the interval $(-1, 1)$ into two disjoint intervals i.e., $\left(-1, \frac{1}{2}\right)$ and $\left(\frac{1}{2}, 1\right)$.

Now, in interval $\left(-1, \frac{1}{2}\right)$, $f'(x) = 2x - 1 < 0$.

Therefore, f is strictly decreasing in interval $\left(-1, \frac{1}{2}\right)$.

However, in interval $\left(\frac{1}{2}, 1\right)$, $f'(x) = 2x - 1 > 0$.

Therefore, f is strictly increasing in interval $\left(\frac{1}{2}, 1\right)$.

Hence, f is neither strictly increasing nor decreasing in interval $(-1, 1)$.

12. Which of the following functions are strictly decreasing on $\left(0, \frac{\pi}{2}\right)$?

(A) $\cos x$ (B) $\cos 2x$ (C) $\cos 3x$ (D) $\tan x$

(A) Let $f_1(x) = \cos x$.

$\therefore f_1'(x) = -\sin x$

In interval $\left(0, \frac{\pi}{2}\right)$, $f_1'(x) = -\sin x < 0$.

$\therefore f_1(x) = \cos x$ is strictly decreasing in interval $\left(0, \frac{\pi}{2}\right)$.

(B) Let $f_2(x) = \cos 2x$.

$\therefore f_2'(x) = -2\sin 2x$

Now, $0 < x < \frac{\pi}{2} \Rightarrow 0 < 2x < \pi \Rightarrow \sin 2x > 0 \Rightarrow -2\sin 2x < 0$

$\therefore f_2'(x) = -2\sin 2x < 0 \; on \left(0, \frac{\pi}{2}\right)$

$\therefore f_2(x) = \cos 2x$ is strictly decreasing in interval $\left(0, \frac{\pi}{2}\right)$.

(C) Let $f_3(x) = \cos 3x$.

$\therefore f_3'(x) = -3\sin 3x$

Now, $f_3'(x) = 0$.

$\Rightarrow \sin 3x = 0 \Rightarrow 3x = \pi, as\ x \in \left(0, \frac{\pi}{2}\right)$

$\Rightarrow x = \frac{\pi}{3}$

The point $x = \frac{\pi}{3}$ divides the interval $\left(0, \frac{\pi}{2}\right)$ into two disjoint intervals

i.e., 0 $\left(0, \frac{\pi}{3}\right)$ and $\left(\frac{\pi}{3}, \frac{\pi}{2}\right)$.

Now, in interval $\left(0, \frac{\pi}{3}\right), f_3(x) = -3\sin 3x < 0 \left[as\ 0 < x < \frac{\pi}{3} \Rightarrow 0 < 3x < \pi\right]$.

$\therefore f_3$ is strictly decreasing in interval $\left(0, \frac{\pi}{3}\right)$.

However, in interval $\left(\frac{\pi}{3}, \frac{\pi}{2}\right), f_3(x) = -3\sin 3x > 0 \left[as\ \frac{\pi}{3} < x < \frac{\pi}{2} \Rightarrow \pi < 3x < \frac{3\pi}{2}\right]$.

$\therefore f_3$ is strictly increasing in interval $\left(\frac{\pi}{3}, \frac{\pi}{2}\right)$.

Hence, f_3 is neither increasing nor decreasing in interval $\left(0, \frac{\pi}{2}\right)$.

(D) Let $f_4(x) = \tan x$.

$\therefore f_4'(x) = \sec^2 x$

In interval $\left(0, \frac{\pi}{2}\right), f_4'(x) = \sec^2 x > 0$.

$\therefore f_4$ is strictly increasing in interval $\left(0, \frac{\pi}{2}\right)$.

Therefore, functions $\cos x$ and $\cos 2x$ are strictly decreasing in $\left(0, \frac{\pi}{2}\right)$.

Hence, the correct answers are A and B.

13. On which of the following intervals is the function f given by $f(x) = x^{100} + \sin x - 1$ strictly decreasing ?

(A) $(0,1)$ (B) $\left(\frac{\pi}{2}, \pi\right)$ (C) $\left(0, \frac{\pi}{2}\right)$ (D) None of these

$f(x) = x^{100} + \sin x - 1$

$\therefore f'(x) = 100x^{99} + \cos x$

In interval $(0, 1)$, $\cos x > 0$ and $100x^{99} > 0$.

$\therefore f'(x) > 0.$

Thus, function f is strictly increasing in interval (0, 1).

In interval $\left(\frac{\pi}{2}, \pi\right)$, $\cos x < 0$ and $100x^{99} > 0$. Also, $100x^{99} > \cos x$.

$\therefore f'(x) > 0$ in $\left(\frac{\pi}{2}, \pi\right)$.

Thus, function f is strictly increasing in interval $\left(\frac{\pi}{2}, \pi\right)$.

In interval $\left(0, \frac{\pi}{2}\right)$, $\cos x > 0$ and $100x^{99} > 0$.

$\therefore 100x^{99} + \cos x > 0$

$\Rightarrow f'(x) > 0$ on $\left(0, \frac{\pi}{2}\right)$

$\therefore f$ is strictly increasing in interval $\left(0, \frac{\pi}{2}\right)$.

Hence, function f is strictly decreasing in none of the intervals.

The correct answer is D.

14. Find the least value of a such that the function f given by $f(x) = x^2 + ax + 1$ is strictly increasing on (1, 2).

$f(x) = x^2 + ax + 1$

$\therefore f'(x) = 2x + a$

Now, function f will be increasing in (1, 2), if $f'(x) > 0$ in (1, 2).

$f'(x) > 0$

$\Rightarrow 2x + a > 0$

$\Rightarrow 2x > -a$

$\Rightarrow x > \frac{-a}{2}$

Therefore, we have to find the least value of a such that

$x > \frac{-a}{2}$, when $x \in (1, 2)$.

$\Rightarrow x > \frac{-a}{2}$ (when $1 < x < 2$)

Thus, the least value of a for f to be increasing on (1, 2) is given by,

$\frac{-a}{2} = 1$

$\frac{-a}{2} = 1 \Rightarrow a = -2$

Hence, the required value of a is -2.

15. Let I be any interval disjoint from (–1, 1). Prove that the function f given by

$$f(x) = x + \frac{1}{x}$$ is strictly increasing on I.

We have,

$$f(x) = x + \frac{1}{x}$$

$$\therefore f'(x) = 1 - \frac{1}{x^2}$$

Now,

$$f'(x) = 0 \Rightarrow \frac{1}{x^2} = 1 \Rightarrow x = \pm 1$$

The points $x = 1$ and $x = -1$ divide the real line in three disjoint intervals i.e., $(-\infty, -1), (-1, 1)$, and $(1, \infty)$

In interval $(-1, 1)$, it is observed that:

$$-1 < x < 1$$
$$\Rightarrow x^2 < 1$$
$$\Rightarrow 1 < \frac{1}{x^2},\ x \neq 0$$
$$\Rightarrow 1 - \frac{1}{x^2} < 0,\ x \neq 0$$

$$\therefore f'(x) = 1 - \frac{1}{x^2} < 0 \text{ on } (-1, 1) \sim \{0\}.$$

$\therefore f$ is strictly decreasing on $(-1, 1) \sim \{0\}$.

In intervals $(-\infty, -1)$ and $(1, \infty)$, it is observed that:

$$x < -1 \text{ or } 1 < x$$
$$\Rightarrow x^2 > 1$$
$$\Rightarrow 1 > \frac{1}{x^2}$$
$$\Rightarrow 1 - \frac{1}{x^2} > 0$$

$$\therefore f'(x) = 1 - \frac{1}{x^2} > 0 \text{ on } (-\infty, -1) \text{ and } (1, \infty).$$

$\therefore f$ is strictly increasing on $(-\infty, 1)$ and $(1, \infty)$.

Hence, function f is strictly increasing in interval **I** disjoint from $(-1, 1)$.

Hence, the given result is proved.

16. Prove that the function f given by $f(x) = \log \sin x$ is strictly increasing on $\left(0, \frac{\pi}{2}\right)$ and strictly decreasing on $\left(\frac{\pi}{2}, \pi\right)$.

We have,

$f(x) = \log \sin x$

$\therefore f'(x) = \frac{1}{\sin x} \cos x = \cot x$

In interval $\left(0, \frac{\pi}{2}\right)$, $f'(x) = \cot x > 0$.

$\therefore f$ is strictly increasing in $\left(0, \frac{\pi}{2}\right)$.

In interval $\left(\frac{\pi}{2}, \pi\right)$, $f'(x) = \cot x < 0$.

$\therefore f$ is strictly decreasing in $\left(\frac{\pi}{2}, \pi\right)$.

17. Prove that the function f given by $f(x) = \log \cos x$ is strictly decreasing on $\left(0, \frac{\pi}{2}\right)$ and strictly increasing on $\left(\frac{\pi}{2}, \pi\right)$.

$f(x) = \log \cos x$

$\therefore f'(x) = \frac{1}{\cos x}(-\sin x) = -\tan x$

In interval $\left(0, \frac{\pi}{2}\right)$, $\tan x > 0 \Rightarrow -\tan x < 0$.

$\therefore f'(x) < 0$ on $\left(0, \frac{\pi}{2}\right)$

$\therefore f$ is strictly decreasing on $\left(0, \frac{\pi}{2}\right)$.

In interval $\left(\frac{\pi}{2}, \pi\right)$, $\tan x < 0 \Rightarrow -\tan x > 0$.

$\therefore f'(x) > 0$ on $\left(\frac{\pi}{2}, \pi\right)$

$\therefore f$ is strictly increasing on $\left(\frac{\pi}{2}, \pi\right)$.

18. Prove that the function given by $f(x) = x^3 - 3x^2 + 3x - 100$ is increasing in **R**.

$$f(x) = x^3 - 3x^2 + 3x - 100$$

$$\begin{aligned} f'(x) &= 3x^2 - 6x + 3 \\ &= 3(x^2 - 2x + 1) \\ &= 3(x-1)^2 \end{aligned}$$

For any $x \in \mathbf{R}$, $(x - 1)^2 > 0$.

Thus, $f'(x)$ is always positive in **R**.

Hence, the given function (f) is increasing in **R**.

19. The interval in which $y = x^2 e^{-x}$ is increasing is

(A) $(-\infty, \infty)$ (B) $(-2, 0)$ (C) $(2, \infty)$ (D) $(0, 2)$

$$y = x^2 e^{-x}$$

$$\therefore \frac{dy}{dx} = 2xe^{-x} - x^2e^{-x} = xe^{-x}(2-x)$$

Now, $\frac{dy}{dx} = 0$.

$\Rightarrow x = 0$ and $x = 2$

The points $x = 0$ and $x = 2$ divide the real line into three disjoint intervals i.e., $(-\infty, 0)$, $(0, 2)$, and $(2, \infty)$.

In intervals $(-\infty, 0)$ and $(2, \infty)$, $f'(x) < 0$ as e^{-x} is always positive.

$\therefore f$ is decreasing on $(-\infty, 0)$ and $(2, \infty)$.

In interval $(0, 2)$, $f'(x) > 0$.

$\therefore f$ is strictly increasing on $(0, 2)$.

Hence, f is strictly increasing in interval $(0, 2)$.

The correct answer is D.