

ASSIGNMENT CLASS XII

(Minimum Learning Level)

RELATIONS AND FUNCTIONS

1. Check whether the relation R in \mathbf{R} defined by $R = \{(a, b) : a \leq b^3\}$ is reflexive, symmetric or transitive.
2. Let $f : \mathbf{R} - \{2\} \rightarrow \mathbf{R} - \{1\}$ is a mapping defined by $f(x) = \frac{x-1}{x-2}$, show that f is bijective.
3. Let $f : \mathbf{N} \rightarrow \mathbf{R}$ be a function defined as $f(x) = 4x^2 + 12x + 15$. Show that $f : \mathbf{N} \rightarrow \mathbf{R}_+$ is invertible. Also find the inverse of f .
4. Show that $f : [-1, 1] \rightarrow \mathbf{R}$, given by $f(x) = \frac{x}{(x+2)}$ is one-one. Find the inverse of the function $f : [-1, 1] \rightarrow \text{Range } f$.
5. Define a binary operation $*$ on the set $\{0, 1, 2, 3, 4, 5\}$ as

$$a * b = \begin{cases} a + b, & \text{if } a + b < 6 \\ a + b - 6 & \text{if } a + b \geq 6 \end{cases}$$

Show that zero is the identity for this operation and each element a of the set is invertible with $6 - a$ being the inverse of a .

6. Show that the operation $*$ on \mathbf{Z} , defined by $a * b = a + b + 1$, satisfies
(i) closure property (ii) associative property (iii) commutative property.
Also find the identity element and inverse of an element $a \in \mathbf{A}$.

INVERSE TRIGONOMETRY

1. Prove that $\tan^{-1} \left(\frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}} \right) = \frac{\pi}{4} - \frac{1}{2} \cos^{-1} x$, $\frac{-1}{\sqrt{2}} \leq x \leq 1$
2. Prove that: (i) $\sin^{-1} \frac{5}{13} + \cos^{-1} \frac{3}{5} = \tan^{-1} \frac{63}{16}$ (ii) $\tan^{-1} \frac{1}{2} + \tan^{-1} \frac{1}{3} = \tan^{-1} \frac{3}{5} + \tan^{-1} \frac{1}{4} = \frac{\pi}{4}$
3. Solve the following equations:
(i) $\tan^{-1}(x+1) + \tan^{-1}(x-1) = \tan^{-1} \frac{6}{17}$ (ii) $\tan^{-1}(x+1) + \tan^{-1}(x-1) = \tan^{-1} \frac{8}{31}$
4. If $\tan^{-1} \frac{x-1}{x-2} + \tan^{-1} \frac{x+1}{x+2} = \frac{\pi}{4}$, then find the value of x

MATRICES AND DETERMINANTS

1. For the matrix $A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$, find λ and μ such that $A^2 + \lambda A + \mu I = 0$. Hence find A^{-1} .

2. Express the given matrix as the sum of symmetric and a skew-symmetric matrix: $\begin{bmatrix} 1 & 3 & 5 \\ -6 & 8 & 3 \\ -4 & 6 & 5 \end{bmatrix}$

3. Find x , if $\begin{bmatrix} x & -5 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$

4. Using properties of determinants prove the following:

$$(i) \begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ x^3 & y^3 & z^3 \end{vmatrix} = xyz(x-y)(y-z)(z-x)$$

$$(ii) \begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc \left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c} \right)$$

$$(iii) \begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$$

$$(iv) \begin{vmatrix} b+c & a & b \\ c+a & c & a \\ a+b & b & c \end{vmatrix} = (a+b+c)(a-c)^2$$

$$(v) \begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$$

5. Using matrix method, solve the following system of linear equations:

$$(i) 2x + y + z = 7$$

$$(ii) x + y + z = 6$$

$$(iii) x + 2y + z = 1$$

$$x - y - z = -4$$

$$x + 2z = 7$$

$$2xy + z = 5$$

$$3x + 2y + z = 10$$

$$3x + y + z = 12$$

$$3x + y - z = 0$$

DIFFERENTIATION

1. Show that the function $f(x) = \begin{cases} \frac{x-|x|}{x}, & x \neq 0 \\ 2 & x = 0 \end{cases}$ is discontinuous at $x=0$.

2. Discuss the continuity and differentiability of $f(x) = |x-1| + |x-2|$.

3. If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$.

4. If $y = x^{\cos x} + \cos x^{\sin x}$, find $\frac{dy}{dx}$.

5. If $x^a y^b = (x+y)^{(a+b)}$, prove that $\frac{dy}{dx} = \frac{y}{x}$.

6. If $y = \tan^{-1} x$, show that $(1+x^2) \frac{d^2 y}{dx^2} + 2x \frac{dy}{dx} = 0$.

7. Find $\frac{d^2 y}{dx^2}$ if $x = a \left(\cos t + \log \tan \frac{t}{2} \right)$, $y = a \sin t$.