

SAMPLE QUESTIONS FOR SECOND INTRA SCHOOL MATHEMATICS OLYMPIAD 2011

CLASS X

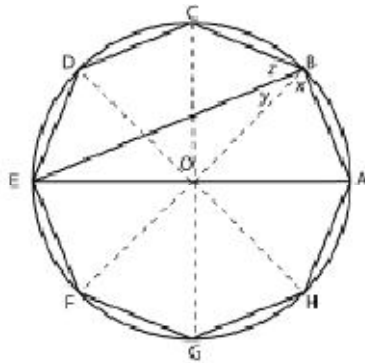
QUESTIONS

1.

How many integer values of x satisfy $\frac{x-2}{5} < \frac{8}{3} < \frac{x+6}{7}$?

2.

In the diagram, what are the measures of the angles x , y and z in the following regular octagon.



3. Shallu was given the task of removing all multiples of 2 and 3 from the set of numbers from 1 to 100. Find the number of numbers that remained.

4. Vicky was writing the page numbers of a book he wrote. If his book has 112 pages, how many digits did Jim write in all?

5.

What is the sum of the solutions of the equation $|2 - |1 - x|| = 1$?

SOLUTIONS

1.

Multiplying the inequation by the product of $3 \times 5 \times 7 = 105$ results in

$$\frac{x-2}{5} < \frac{8}{3} < \frac{x+6}{7}$$

$$\frac{(105)(x-2)}{5} < \frac{(105)(8)}{3} < \frac{(105)(x+6)}{7}$$

$$21(x-2) < 280 < 15(x+6)$$

In order to solve the inequation then,

$$21x - 42 < 280 \quad \text{and} \quad 15x + 90 > 280 \quad \text{and} \quad 21x - 42 < 15x + 90$$

$$21x < 322 \quad \text{and} \quad 15x > 190 \quad \text{and} \quad 6x < 132$$

$$x < 15\frac{1}{3} \quad \text{and} \quad x > 12\frac{2}{3} \quad \text{and} \quad x < 22$$

Therefore, the only integers that satisfy all three conditions are 13, 14, and 15.

2.

Since the shape is a regular octagon then

$$\angle AOB = \frac{360^\circ}{8} = 45^\circ \text{ and } \triangle AOB \text{ is an isosceles}$$

triangle so $\angle OBA = \angle OAB$. Thus angle

$$x = \frac{180^\circ - 45^\circ}{2} = 67.5^\circ. \text{ Also using the property that}$$

all inscribed angles subtended by a diameter are right angles, we determine that

$$y = 90^\circ - x = 90^\circ - 67.5^\circ = 22.5^\circ. \text{ Another way to}$$

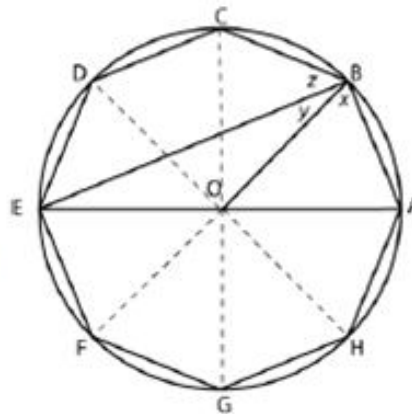
find y is to notice that $\angle EOB = 180^\circ - 45^\circ = 135^\circ$ and $\triangle EOB$ is isosceles. Thus

$$y = \frac{180^\circ - 135^\circ}{2} = 22.5^\circ.$$

Since $\triangle COB$ is congruent to $\triangle BOA$ then $\angle OBC = \angle OBA$. Thus $x = y + z$ and since

$$x = 67.5^\circ \text{ and } y = 22.5^\circ \text{ then } z = 67.5^\circ - 22.5^\circ = 45^\circ.$$

Alternatively, it can be determined that $\angle CBA = 135^\circ$. Since $x + y = 90^\circ$ then $z = 45^\circ$.



3. There are 50 multiples of 2 from 1 to 100, both numbers included, namely 2, 4, 6, etc. There are 33 multiples of 3 from 1 to 100, both numbers included, namely 3, 6, 9, etc. When added up, 83 numbers were counted, of which some were counted twice, namely all the numbers which are both multiples of 2 and 3, e.g. 6, 12, 18, etc. The numbers that are both multiples of 2 and 3, at the same time, are also multiples of 6, therefore if the number of multiples of 6 are removed from the above number once, it will cover the doubles. There are 16 multiples of 6 from 1 to 100, both numbers included, namely 6, 12, 18, etc. Therefore $83 - 16 = 67$ numbers have been removed and 33 remained.

4. No. from 1, 2, ..., 9 : sum of digits = $1 + 2 + \dots + 9 = 45$

$$\text{No. from 10, 11, ..., 19 sum of digits} = 1+0 + 1+1 + \dots + 1+9 = 1 \cdot 10 + 45$$

$$\text{Similarly, } 90 + 91 + \dots + 99 \text{ sum of digits} = 9 \cdot 10 + 45$$

$$\text{So, sum till here} = 45 \cdot 10 + (10 + 20 + \dots + 90) = 450 + 450 = 900$$

Now, 100, 101, ... 112 sum of digits =

$$(1+0+0) + (1+0+1) + \dots + (1+1+2) =$$

$$(1+2+\dots+10) + (1+1) + (1+1+1) + (1+1+2) = 55 + 9 = 64.$$

So, total sum = $900 + 64 = 964$.

5.

To satisfy the equation, $2 - |1 - x| = 1$ or -1 . Correspondingly, $|1 - x| = 1$ or $|1 - x| = 3$.

The first equation is satisfied by $x = 0$ and $x = 2$, while the second by $x = -2$ and $x = 4$. The sum of these four solutions is 4.