COMPARTMENT PAPER

(BY CBSE) HELD IN JULY 2010

MATHEMATICS

Class XII

Time : 3 Hours **Max. Marks : 100**

General Instructions:

1. All questions are compulsory.
2. The question paper consist of 29 questions divided into three sections A, B and C. Section A comprises of 10 questions of one mark each, section B comprises of 12 questions of four marks each and section C comprises of 07 questions of six marks each.
3. All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
4. There is no overall choice. However, Internal choice has been provided in 04 questions of four marks each and 02 questions of six marks each. You have to attempt only one of the alternatives in all such questions.
5. Use of calculators is not permitted. You may ask for logarithmic tables, if required.

SECTION – A

1. If $f : R \to R$ is defined by $f(x) = 3x + 2$, find $f(f(x))$.
2. If $\sin^{-1}\left(\frac{1}{3}\right) + \cos^{-1} x = \frac{\pi}{2}$, then find x.
3. If $\begin{pmatrix} a+b & 2 \\ 5 & b \end{pmatrix} = \begin{pmatrix} 6 & 5 \\ 2 & 2 \end{pmatrix}^T$, then find a.
4. If A is a matrix of order 3×4 and B is a matrix of order 4×3, find the order of the matrix (AB).
5. If $A = \begin{pmatrix} 3 & 1 \\ 2 & -3 \end{pmatrix}$, then find $|adj\ A|$.
6. Evaluate: $\int \frac{x^3 - 1}{x^2} dx$
7. Evaluate: $\int_{-\pi/4}^{\pi/4} \sin^3 x\, dx$
8. Find a vector in the direction of $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$, which has magnitude 6 units.
9. Find the position vector of the mid-point of the line-segment AB, where A is the point $(3, 4, -2)$ and B is the point $(1, 2, 4)$.
10. Find the distance of the point $(2, 3, 4)$ from the x-axis.

SECTION – B

11. Show that the function $f:R\to R$ given by $f(x)=ax+b$, where $a,b\in R, a\neq 0$ is a bijection.

12. Prove the following:

$$\cos^{-1}\left(\frac{4}{5}\right)+\cos^{-1}\left(\frac{12}{13}\right)=\cos^{-1}\left(\frac{33}{65}\right)$$

OR

Solve for x:

$$\tan^{-1}(x+2)+\tan^{-1}(x-2)=\tan^{-1}\left(\frac{8}{79}\right)\ ;\ x>0$$

13. Using properties of determinants, prove the following:

$$\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ x^3 & y^3 & z^3 \end{vmatrix}=xyz(x-y)(y-z)(z-x)$$

14. For what value of k is the function defined by $f(x)=\begin{cases} k(x^2+2), & if\ x\le 0 \\ 3x+1, & if\ x>0 \end{cases}$ continous at $x=0$?

Also write whether the function is continous at $x=1$.

15. If $y=\left(\cot^{-1}x\right)^2$, then sow that $\left(x^2+1\right)^2\frac{d^2y}{dx^2}+2x\left(x^2+1\right)\frac{dy}{dx}=2$

OR

If $x^y=e^{x-y}$, then prove that $\frac{dy}{dx}=\frac{\log x}{(1+\log x)^2}$

16. Find the intervals on which the following function is (a) increasing (b) decreasing:

$$f(x)=2x^3-9x^2+12x+15$$

OR

Find the equation of the tangent to the curve $y=\frac{x-7}{x^2-5x+6}$ at the point where it cuts the x-axis.

17. Evaluate: $\int\left[\log(\log x)+\frac{1}{(\log x)^2}\right]dx$ **OR** $\int\frac{dx}{\left(x^2+1\right)\left(x^2+2\right)}$

18. Find the differential equation of the family of all circles touching the x-axis at the origin.

19. Solve the following differential equation:

$$xy\log\left(\frac{y}{x}\right)dx+\left(y^2-x^2\log\left(\frac{y}{x}\right)\right)dy=0$$

20. Using vectors, find the area of the triangle with vertices, $A(2,3,5)$, $B(3,5,8)$ and $C(2,7,8)$.

21. Find the equation of the plane passing through the point $A(1,2,1)$ and perpendicular to the line joining the points $P(1,4,2)$ and $Q(2,3,5)$. Also, find the distance of this plane from the line $\frac{x+2}{2}=\frac{y-5}{-1}=\frac{z-7}{-1}$.

22. Find the probability distribution of the number of doublets in three throws of a pair of dice and hence find its mean.

SECTION – C

23. If $A=\begin{pmatrix} 3 & 2 & 1 \\ 4 & -1 & 2 \\ 7 & 3 & -3 \end{pmatrix}$, then find A^{-1}. Hence solve the following system of equations:

$3x+2y+z=6$; $4x-y+2z=5$; $7x+3y-3z=7$

24. Find the maximum area of an isosceles triangle inscribed in the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$, with its vertex at one end of the major axis.

25. Using integration, find the area of the region $\{(x,y): x^2+y^2 \le 16,\ x^2 \le 6y\}$.

OR

Find the area of the region $\{(x,y): x^2+y^2 \le 1 \le x+y\}$ by using integration.

26. Evaluate $\int_1^2 \left(x^2+5x\right) dx$ as limit of sums.

OR

Evaluate: $\int \left(\sqrt{\tan x}+\sqrt{\cot x}\right) dx$

27. Find the image of the point $(1,6,3)$ in the line $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$. Also write the equation of the line joining the given point and its image and find the length of the segment joining the given point and its image.

28. A dealer deals in two items A and B. He has Rs. 15,000 to invest and a space to store at the most 80 pieces. Item A costs him Rs. 300 and item B costs him Rs. 150. He can sell items A and B at profits of Rs. 40 and Rs. 25 respectively. Assuming that he can sell all that he buys, formulate the above as a linear programming problem for maximum profit and solve it graphically.

29. In a bolt factory machines, A, B and C manufacture respectively 25%, 35% and 40% of the total bolts. Of their output 5, 4 and 2 percent are respectively defective bolts. A bolt is drawn at random from producton and is found to be defective. What is the probability that it manufactured by the machine B.

******************************* END OF PAPER ********************************

QUESTIONS FROM SET B AND C WHICH WERE NOT IN COMMON WITH SET A

SET B/C (4 MARKERS)

1. Prove the following: (a) $\sin^{-1}\frac{3}{5}+\sin^{-1}\frac{8}{17}=\cos^{-1}\frac{36}{85}$ (b) $\sin^{-1}\frac{3}{5}+\cos^{-1}\frac{12}{13}=\sin^{-1}\frac{56}{65}$
2. Solve for x: (a) $\tan^{-1}\frac{x}{2}+\tan^{-1}\frac{x}{3}=\frac{\pi}{4}$; $\sqrt{6}>x>0$ (b) $2\tan^{-1}(\sin x)=\tan^{-1}(2\sec x)$
3. Using properties of determinants, prove the following:

(a) $\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ y+z & z+x & x+y \end{vmatrix}=(x-y)(y-z)(z-x)(x+y+z)$ (b) $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix}=abc+bc+ca+ab$

4. Find the value of a and b such that the function defined as follows is continuous:

$$f(x)=\begin{cases} x+2; & x\le 2 \\ ax+b; & 2<x<5 \\ 3x-2; & x\ge 5 \end{cases}$$

5. Find the value of k, for which the function f defined below is continuous:

$$f(x)=\begin{cases} \frac{k\cos x}{\pi-2x}, & x<\frac{\pi}{2} \\ 3 & x=\frac{\pi}{2} \\ \frac{3\tan 2x}{2x-\pi} & x>\frac{\pi}{2} \end{cases}$$

SET B/C (6 MARKERS)

1. Find $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ Use this to solve the equations $\begin{matrix} x-y+z=4 \\ x-2y-2z=9 \\ 2x+y+3z=1 \end{matrix}$.

2. If $A=\begin{pmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{pmatrix}$, find A^{-1}. Hence solve the following system of equations:

$$x+2y-3z=-4\ ;\ 2x+3y+2z=2\ ;\ 3x-3y-4z=11$$

3. Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere.

4. Prove that the radius of the right circular cylinder of greatest curved surface area, which can be inscribed in a given right circular cone, is half that of the cone.