RE-TEST
FIRST TERMINAL EXAMINATION 2011-12
MATHEMATICS
Class XII

Time : 3 Hours **Max. Marks : 100**

General Instructions:

1. All questions are compulsory.
2. The question paper consist of 29 questions divided into three sections A, B and C. Section A comprises of 10 questions of one mark each, section B comprises of 12 questions of four marks each and section C comprises of 07 questions of six marks each.
3. All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
4. There is no overall choice. However, Internal choice has been provided in 04 questions of four marks each and 02 questions of six marks each. You have to attempt only one of the alternatives in all such questions.
5. Use of calculators is not permitted. You may ask for logarithmic tables, if required.

SECTION – A

1. Write gof when and $f: R \rightarrow R$ and $g: R \rightarrow R$ are defined by: $f(x) = 2x + 3$ and $g(x) = x^2 + 5$.

2. Is the function $f: R \rightarrow R$, given by $f(x) = |x|$, one-one or onto. Justify.

3. If $\sin^{-1}\left(\frac{1}{3}\right) + \cos^{-1} x = \frac{\pi}{2}$, then find x.

4. Write the value of $\cos\left[\tan^{-1}\left(\frac{3}{4}\right)\right]$.

5. If $\begin{pmatrix} a+b & 2 \\ 5 & b \end{pmatrix} = \begin{pmatrix} 6 & 5 \\ 2 & 2 \end{pmatrix}^T$, then find a.

6. If $0 < x < \pi$ and the matrix $\begin{bmatrix} 2\sin x & 3 \\ 1 & 2\sin x \end{bmatrix}$ is singular, write the value(s) of x.

7. Write all the points of discontinuity(if any) of f defined by $f(x) = |x-1| + |x+2|$.

8. Is Rolle's theorem applicable to $f(x) = x^2 - 4$ for $x \in [1, 2]$? Justify.

9. If $y = \tan^{-1}(\cot x)$, write value of dy/dx.

10. What is the value of the integral : $\int \frac{x^3 - 1}{x^2} dx$

SECTION – B

11. Let S be the set of all real numbers and let R be a relation in S, defined by $R=\{(a,b):a\le b^2\}$. Show that R satisfies none of reflexivity, symmetry and transitivity.

OR

Define a binary operation $*$ on the set {0, 1, 2, 3, 4, 5} as $a*b=\begin{cases} a+b & \text{if } a+b<6 \\ a+b-6 & \text{if } a+b\ge 6 \end{cases}$

Show that zero is the identity for this operation and each element *a* of the set is invertible with 6 – *a* being the inverse of *a*.

12. Prove that: $\cot^{-1}\left(\dfrac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)=\dfrac{x}{2} \quad ; x\in\left(0,\dfrac{\pi}{4}\right)$

13. Express the given matrix as the sum of symmetric and a skew-symmetric matrix:

$$\begin{bmatrix} 1 & 3 & 5 \\ -6 & 8 & 3 \\ -4 & 6 & 5 \end{bmatrix}$$

14. If $y=\dfrac{\sin^{-1}x}{\sqrt{1-x^2}}$, prove that $(1-x^2)\dfrac{dy}{dx}=xy+1$.

OR

If $x\sqrt{1+y}\ +\ y\sqrt{1+x}\ =\ 0$ for $-1<x<1$, prove that $\dfrac{dy}{dx}=-\dfrac{1}{(1+x)^2}$

15. If $y=(\sin x)^x+x^{\log x}$, find $\dfrac{dy}{dx}$.

16. If $x=a(\cos t+t\sin t)$ and $y=a(\sin t-t\cos t)$, find $\left(\dfrac{d^2y}{dx^2}\right)$.

17. Using differentials, find the approximate value of $(29)^{1/3}$ up to 2 places of decimal.

18. Find equation of the tangent to the curve $y=4x^3-3x+4$, which are perpendicular to $9y+x+3=0$.

OR

Sand is pouring from a pipe at the rate of $12\,cm^3/s$. The falling sand forms a cone on the ground in such a way that the height of the cone is always one sixth of the radius of the base. How fast is the height of the sand cone increasing when the height is 5 cm ?

19. Find the absolute maximum value and absolute minimum value of the following function:

$$f(x)=4x-\frac{x^2}{2} \text{ in } [-2, 4.5]$$

20. Evaluate the following integral:

$$\int \sin^4 x \, dx$$

OR

$$\int \frac{1}{1+\cot x} dx$$

21. Evaluate the following integral:

$$\int \frac{4x-3}{x^2+3x-10} dx$$

22. A dietician wishes to mix two types of foods in such a way that vitamin contents of the mixture contain atleast 8 units of vitamin A and 10 units of vitamin C. Food 'I' contains 2 units/kg of vitamin A and 1 unit/kg of vitamin C. Food 'II' contains 1 unit/kg of vitamin A and 2 units/kg of vitamin C. It costs Rs 50 per kg to purchase Food 'I' and Rs 70 per kg to purchase Food 'II'. Formulate this problem as a linear programming problem to minimise the cost of such a mixture.

(Do not solve it graphically)

SECTION – C

23. Let $f: R-\{2\} \rightarrow R-\{1\}$ is a mapping defined by $f(x)=\frac{x-1}{x-2}$, show that f is bijective. Hence find the inverse of f.

24. Solve the following equation:

$$\tan^{-1}(x+2) + \tan^{-1}(x-2) = \tan^{-1}\left(\frac{8}{79}\right) \; ; \; x>0$$

25. Using matrix method, solve the following system of linear equations:

$$2x+y+z=7$$
$$x-y-z=-4$$
$$3x+2y+z=10$$

26. Using properties of determinants, show that :

$$\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ y+z & z+x & x+y \end{vmatrix} = (x-y)(y-z)(z-x)(x+y+z).$$

OR

Using properties of determinants, solve for x :

$$\begin{vmatrix} 3x-8 & 3 & 3 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8 \end{vmatrix} = 0$$

27. Show that the height of the closed right circular cylinder of given volume and minimum total surface area , is equal to its diameter.

OR

A window is in the form of an rectangle surmounted by a semi-circular opening. The total perimeter of the window is 10m. Find the dimensions of the window to admit maximum light through the whole opening.

28. Find the intervals in which the function *f* given by: $f(x) = (x-1)(x-2)^2$ is:

(i) strictly increasing (ii) strictly decreasing.

29. A furniture firm manufactures chairs and tables, each requiring the use of three machines A, B and C. Production of one chair requires 2 hours on machine A, 1 hour on machine B and 1 hour on machine C. Each table requires 1 hour each on machine A and B and 3 hours on machine C. The profit obtained by selling one chair is Rs. 30 while by selling one table the profit is Rs. 60. The total time available per week on machine A is 70 hours, on machine B is 40 hours and on machine C is 90 hours. How many chairs and tables should be made per week so as to maximize profit? Formulate the problem as L.P.P. and solve it graphically.