FIRST TERMINAL EXAMINATION 2011-12
MATHEMATICS
Class XII

Time : 3 Hours **Max. Marks : 100**

General Instructions:

1. All questions are compulsory.
2. The question paper consist of 29 questions divided into three sections A, B and C. Section A comprises of 10 questions of one mark each, section B comprises of 12 questions of four marks each and section C comprises of 07 questions of six marks each.
3. All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
4. There is no overall choice. However, Internal choice has been provided in 04 questions of four marks each and 02 questions of six marks each. You have to attempt only one of the alternatives in all such questions.
5. Use of calculators is not permitted. You may ask for logarithmic tables, if required.

SECTION – A

1. If $f : R \to R$ is defined by $f(x) = 3x - 2$, find $f(f(x))$.

2. Is the function $f : R \to R$, given by $f(x) = [x]$, one-one or onto, where $[x]$ denotes the greatest integer less than or equal to x. Justify.

3. What is the principal value of $\cos^{-1}\left(\cos\frac{2\pi}{3}\right) + \sin^{-1}\left(\sin\frac{2\pi}{3}\right)$?

4. Express $\cot^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right)$; $|x| > 1$ in the simplest form.

5. If A is a matrix of order 3×4 and B is a matrix of order 4×3, find order of the matrix (AB).

6. A is a square matrix of order 3 and $|A| = 4$. Write the value of $|adj\ A|$.

7. Find $\left.\frac{dy}{dx}\right|_{x=\pi/2}$ where $y = e^{\sin x}$.

8. What are the maximum and minimum values (if exist) of the function $f(x) = -|x+4| + 3$.

9. What is the slope of the normal to the curve $y = x^3 - 4x$ at $x = 3$?

10. What is the value of the integral $\int \frac{\cos 2x}{\sin^2 x \cos^2 x} dx$.

SECTION – B

11. Show that the relation R on the set Z of integers given by $R=\{(a,b): 4 \text{ divides } a-b\}$, is an equivalence relation.

OR

Show that the operation $*$ on Z, defined by $a*b=a+b+1$, satisfies commutative and associative property. Also find the identity element and inverse of an element $a \in A$.

12. Prove that: $\sin^{-1}\frac{5}{13}+\cos^{-1}\frac{3}{5}=\tan^{-1}\frac{63}{16}$.

13. Using elementary transformations, find the inverse of the following matrix(if it exists):

$$A=\begin{bmatrix}4 & 5\\ 3 & 4\end{bmatrix}$$

14. If the function $f(x)=\begin{cases}3ax+b & \text{for} \quad x>1\\ 11 & \text{for} \quad x=1\\ 5ax-2b & \text{for} \quad x<1\end{cases}$, is continous at $x=1$, find the values of a and b.

OR

If $y=e^x(\sin x+\cos x)$, prove that $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+2y=0$.

15. Find $\frac{dy}{dx}$ if $y=(\cos x)^{\log x}+a^{\left(x+\frac{1}{x}\right)}$.

16. Find $\left.\frac{d^2y}{dx^2}\right|_{\theta=\pi/4}$ if $x=a(\theta-\sin\theta)$, $y=b(1+\cos\theta)$.

17. Using differentials, find the approximate value of $(32.15)^{1/5}$ up to 2 places of decimal.

18. Find the absolute maximum and minimum values of a function f given by:

$f(x)=2x^3-15x^2+36x+1$ on the interval $[1, 5]$.

19. Show that the curves $2x = y^2$ and $2xy = k$ cut at right angles if $k^2 = 8$.

OR

A balloon, which always remains spherical on inflation, is being inflated by pumping in 900 cm^3 of gas per second. Find the rate at which the radius of the balloon increases when the radius is 10 cm.

20. Evaluate the following integral:

$$\int \cos^4 x \, dx$$

OR

$$\int \frac{1}{1-\tan x} dx$$

21. Evaluate the following integral:

$$\int \frac{x+2}{\sqrt{x^2+2x+3}} dx$$

22. A toy company manufactures two types of dolls, A and B. Market tests and available resources have indicated that the combined production level should not exceed 1200 dolls per week and the demand for dolls of type B is at most half of that for dolls of type A. Further, the production level of dolls of type A can exceed three times the production of dolls of other type by at most 600 units. If the company makes profit of Rs 12 and Rs 16 per doll respectively on dolls A and B, how many of each should be produced weekly? Formulate this problem as a linear programming problem in order to maximise the profit. (Do not solve it graphically)

SECTION – C

23. Consider $f : R_+ \to [-5, \infty)$ given by $f(x) = 9x^2 + 6x - 5$. Show that f is invertible with

$$f^{-1}(y) = \frac{\sqrt{y+6}-1}{3}.$$

24. Solve the following equation:

$$\tan^{-1}(x+1) + \tan^{-1}(x-1) = \tan^{-1}\frac{8}{31}$$

25. Using matrix method, solve the following system of linear equation:

$$x+y+z=6$$
$$x+2z=7$$
$$3x+y+z=12$$

26. Using properties of determinants, show that :

$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = \left(1+a^2+b^2\right)^3$$

OR

If x, y, z are different and $\Delta = \begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$, then show that $1+xyz=0.$

27. Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere.

OR

A rectangular sheet of tin 45 cm by 24 cm is to be made into a box without top, by cutting off square from each corner and folding up the flaps. What should be the side of the square to be cut off so that the volume of the box is maximum ?

28. Find the intervals in which the function *f* given by: $f(x) = \sin x + \cos x \quad ; \ 0 \le x \le 2\pi$ is:

(i) strictly increasing (ii) strictly decreasing.

29. A dealer deals in two items A and B. He has Rs. 15,000 to invest and a space to store at the most 80 pieces. Item A costs him Rs. 300 and item B costs him Rs. 150. He can sell items A and B at profits of Rs. 40 and Rs. 25 respectively. Assuming that he can sell all that he buys, formulate the above as a linear programming problem for maximum profit and solve it graphically.