ANSWER KEY Class XII C Differentiation (held on 02/08/11)

Ans 1 We know composition of continuous functions is continuous. Since $\sin|x^2|$ is composition of three continuous fns. ie. trigonometric, polynomial and modulus $\therefore$ it is continuous.

Ans 2 Put $x = \tan\theta \Rightarrow \theta = \tan^{-1}x$

$$\therefore\ y = \cos^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right) = \cos^{-1}(\sin 2\theta) = \cos^{-1}\left(\cos\left(\frac{\pi}{2}-2\theta\right)\right) = \frac{\pi}{2} - 2\theta$$

$$= \frac{\pi}{2} - 2\tan^{-1}x \qquad \therefore \frac{dy}{dx} = \frac{-2}{1+x^2}$$

Ans 3 Since $[x]$ is not continuous and diff, $\therefore$ Rolle's theorem not applicable

Ans 4 $y\log x = x - y$

$\Rightarrow y + y\log x = x$

$\Rightarrow y(1+\log x) = x \Rightarrow y = \dfrac{x}{1+\log x}$

$$\therefore \frac{dy}{dx} = \frac{(1+\log x)(1) - x\left(\frac{1}{x}\right)}{(1+\log x)^2} = \frac{\log x}{(1+\log x)^2} \quad \text{Hence proved.}$$

OR

$x\sqrt{1+y} + y\sqrt{1+x} = 0$

$\Rightarrow x\sqrt{1+y} = -y\sqrt{1+x}$ squaring,

$x^2(1+y) = y^2(1+x)$

$\Rightarrow x^2 + x^2y = y^2 + y^2x$

$\Rightarrow x^2 - y^2 + x^2y - y^2x = 0$

$\Rightarrow (x-y)(x+y) + xy(x-y) = 0$

$\Rightarrow (x-y)(x+y+xy) = 0$

$\Rightarrow x + y + xy = 0 \qquad (\because x - y \neq 0)$

$\Rightarrow x + y(1+x) = 0 \Rightarrow y = \dfrac{-x}{1+x}$

$$\therefore \frac{dy}{dx} = \frac{(1+x)(-1) - (-x)(+1)}{(1+x)^2} = \frac{-1}{(1+x)^2} \quad \text{Hence proved}$$

Ans 5 Let $y = u + v$ where $u = \left(x+\frac{1}{x}\right)^x$, $v = (\log x)^{\cos x}$

$$\therefore \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \quad \text{--- (1)}$$

Now, $u = \left(x+\frac{1}{x}\right)^x$

taking log both sides,

$$\log u = \log\left(x+\frac{1}{x}\right)^x$$

$$\Rightarrow \log u = x \cdot \log\left(x+\frac{1}{x}\right)$$

diff. both sides wrt x,

$$\frac{1}{u}\frac{du}{dx} = x \cdot \frac{1}{\left(x+\frac{1}{x}\right)} \cdot \left(1-\frac{1}{x^2}\right) + \log\left(x+\frac{1}{x}\right) \cdot 1$$

$$= \frac{x^2}{x^2+1} \times \frac{x^2-1}{x^2} + \log\left(x+\frac{1}{x}\right)$$

$$= \frac{x^2-1}{x^2+1} + \log\left(x+\frac{1}{x}\right)$$

$$\Rightarrow \frac{du}{dx} = \left(x+\frac{1}{x}\right)^x \left[\frac{x^2-1}{x^2+1} + \log\left(x+\frac{1}{x}\right)\right] \quad \text{—— } (2)$$

Again, $v = (\log x)^{\cos x}$

taking log both sides,

$$\log v = \cos x \cdot \log(\log x)$$

diff. both sides wrt x,

$$\frac{1}{v}\frac{dv}{dx} = \cos x \cdot \frac{1}{\log x} \cdot \frac{1}{x} + \log(\log x) \cdot (-\sin x)$$

$$\Rightarrow \frac{dv}{dx} = (\log x)^{\cos x}\left[\frac{\cos x}{x \log x} - \sin x \cdot \log(\log x)\right] \quad \text{—— } (3)$$

from (1), (2) and (3),

$$\frac{dy}{dx} = \left(x+\frac{1}{x}\right)^x\left[\frac{x^2-1}{x^2+1} + \log\left(x+\frac{1}{x}\right)\right] + (\log x)^{\cos x}\left[\frac{\cos x}{x \log x} - \sin x \cdot \log(\sin x)\right] \text{ Ans}$$

Ans 6 $x = a(\theta - \sin\theta) \Rightarrow \dfrac{dx}{d\theta} = a(1-\cos\theta)$

$y = a(1+\cos\theta) \Rightarrow \dfrac{dy}{d\theta} = -a\sin\theta$

Now, $$\frac{dy}{dx} = \frac{\left(\frac{dy}{d\theta}\right)}{\left(\frac{dx}{d\theta}\right)} = \frac{-a\sin\theta}{a(1-\cos\theta)} = -\frac{2\sin\theta/2\cos\theta/2}{2\sin^2\theta/2} = -\cot\theta/2$$

Now, $$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(-\cot\frac{\theta}{2}\right) = -\left(-\operatorname{cosec}^2\theta/2\right) \times \frac{1}{2} \times \frac{d\theta}{dx}$$

$$= \frac{1}{2}\operatorname{cosec}^2\theta/2 \times \frac{1}{a(1-\cos\theta)}$$

$$= \frac{1}{2a} \cdot \operatorname{cosec}^2\theta/2 \times \frac{1}{2\sin^2\theta/2}$$

$$= \frac{1}{4a}\operatorname{cosec}^4\theta/2 \text{ Ans.}$$