# Answer Key class Test XII BC Differentiation

**Ans 1** (a) $\frac{dy}{dx} = 3^{\tan x} \cdot \log_e 3 \cdot \sec^2 x$

(b) $y = \cos^{-1}(\sin x) = \cos^{-1}\left(\cos\left(\frac{\pi}{2} - x\right)\right) = \frac{\pi}{2} - x \quad \therefore \frac{dy}{dx} = -1$

(c) $y = (a^2 - x^2)^{-1/2} \Rightarrow \frac{dy}{dx} = -\frac{1}{2}(a^2 - x^2)^{-1/2 - 1} \cdot (-2x) = \frac{x}{(a^2 - x^2)^{3/2}}$

(d) $y = \frac{1 + \tan x}{1 - \tan x} = \tan\left(\frac{\pi}{4} + x\right) \Rightarrow \frac{dy}{dx} = \sec^2\left(\frac{\pi}{4} + x\right)$

(e) $\frac{dy}{dx} = -\sin(\tan^{-1} e^{-x}) \frac{1}{1 + (e^{-x})^2} \cdot e^{-x} \cdot (-1) = \frac{e^{-x} \cdot \sin(\tan^{-1} e^{-x})}{1 + e^{-2x}}$

**Ans 2** clearly, when $x < 2$, being a polynomial, f is continous.
when $x > 2$, again being polynomial, f is continous.
Possible point of discontinuity is $x = 2$.

At $x = 2$

LHL $= \lim_{h \to 0} f(2 - h) = \lim_{h \to 0} (2-h)^3 - 3 = 5$

RHL $= \lim_{h \to 0} f(2 + h) = \lim_{h \to 0} (2+h)^2 + 1 = 5$

also, $f(2) = 2^3 - 5 = 5$.

Since, LHL $=$ RHL $= f(2) \therefore$ f is continous everywhere.

**OR**

Since f is continous, it is continous at $x = 2$ and 10 also.

Now, f is continous at $x = 2$

$\Rightarrow \lim_{x \to 2} f(x) = f(2)$

$\Rightarrow \lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x) = f(2)$

$\Rightarrow \lim_{h \to 0} f(2-h) = \lim_{h \to 0} f(2+h) = f(2)$

$\Rightarrow \lim_{h \to 0} 5 = \lim_{h \to 0} a(2+h) + b = 5 \Rightarrow 2a + b = 5$ —— (1)

Again, f is cts. at $x = 10$.

$\Rightarrow \lim_{h \to 0} f(10 - h) = \lim_{h \to 0} f(10 + h) = f(10)$

$\Rightarrow \lim_{h \to 0} [a(10 - h) + b] = \lim_{h \to 0} 21 = 21$

$\Rightarrow 10a + b = 21$ —— (2)

Solving (1) and (2),

$a = 2$ and $b = 1$ Ans.

**Ans 3** $f(x) = \begin{cases} -(x+1)-(x-2) & x \leq -1 \\ (x+1)-(x-2) & -1 < x < 2 \\ (x+1)+(x-2) & x \geqslant 2 \end{cases}$


$$= \begin{cases} -2x+1 & x \leq -1 \\ 3 & -1 < x < 2 \\ 2x-1 & x \geqslant 2 \end{cases}$$

clearly f being polynomial fn. is diff. everywhere except possible points of discontinuity at $x = -1$ and $x = 2$.

At $x = -1$

$$LHD = \lim_{h \to 0} \frac{f(-1-h) - f(-1)}{-h} = \lim_{h \to 0} \frac{[2(-1-h)+1] - 3}{-h} = \lim_{h \to 0} \frac{2h}{-h} = -2$$

$$RHD = \lim_{h \to 0} \frac{f(-1+h) - f(-1)}{h} = \lim_{h \to 0} \frac{3-3}{-h} = 0$$

$\Rightarrow$ f is not derivable at $x = -1$.

At $x = 2$

$$LHD = \lim_{h \to 0} \frac{f(2-h) - f(2)}{-h} = \lim_{h \to 0} \frac{3-3}{-h} = 0$$

$$RHD = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h} = \lim_{h \to 0} \frac{[2(2+h)-1] - 3}{h} = \lim_{h \to 0} \frac{2h}{h} = 2$$

$\Rightarrow$ f is not derivable at $x = 2$.

Again, we know modulus fn. is continous everywhere, and sum of two continous fn. is also continous.

$\therefore$ f is continous everywhere, but not diff. at $x = -1$ and $x = 2$.

**Ans 4** (a) $2\sin x . \cos x + 2\cos y . (-\sin y) . \frac{dy}{dx} = 0$

$\Rightarrow \sin 2x = \sin 2y . \frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx} = \frac{\sin 2x}{\sin 2y}$ Ans.

(b) $y = \frac{\pi}{2} - \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = \frac{\pi}{2} - 2\tan^{-1}x$

$\therefore \frac{dy}{dx} = \frac{-2}{1+x^2}$ Ans.