Answer Key XII A test

Ans 1 (a) $4^{\cot x} \times (-\csc^2 x) \times \log_e 4$

(b) $y = \cot^{-1}\left(\cot\left(\frac{\pi}{2} - x\right)\right) = \frac{\pi}{2} - x \quad \therefore \frac{dy}{dx} = -1$

(c) $y' = \frac{1}{x + \sqrt{1+x^2}}\left(1 + \frac{\cancel{2}x}{\cancel{2}\sqrt{1+x^2}}\right) = \frac{1}{\cancel{x+\sqrt{1+x^2}}}\left(\frac{\cancel{\sqrt{1+x^2}+x}}{\sqrt{1+x^2}}\right) = \frac{1}{\sqrt{1+x^2}}$

(d) $y = \tan^{-1}\left(\frac{1+\tan x}{1-\tan x}\right) = \tan^{-1}\left(\tan\left(\frac{\pi}{4}+x\right)\right) = \frac{\pi}{4} + x \Rightarrow \frac{dy}{dx} = 1$

(e) $\frac{dy}{dx} = \sec^2(\cos^{-1} e^{-x}) \cdot \frac{-1}{\sqrt{1-(e^{-x})^2}} \times e^{-x} \times -1$

$= \frac{e^{-x}}{\sqrt{1-e^{-2x}}} \sec^2(\cos^{-1} e^{-x})$

Ans 2 when $x \le -3$, being a sum of modulus and constant functions which are always continous, f is continous.

Similar cases for $-3 < x < 3$ and $x \ge 3$ can be discussed.

Possible point of discountinuity are -3 and 3.

At $x = -3$

LHL $= \lim_{h \to 0} f(-3-h) = \lim_{h \to 0} |-3-h| + 3 = 6$

RHL $= \lim_{h \to 0} f(-3+h) = \lim_{h \to 0} -2(-3+h) = 6$.

Also, $f(-3) = |-3| + 3 = 6 \quad \therefore$ continous at $x = -3$.

At $x = 3$

LHL $= \lim_{h \to 0} f(3-h) = -2(3-h) = -6$

RHL $= \lim_{h \to 0} (3+h) = 6(3+h) + 2 = 20$. Also, $f(3) = 20$

Since, LHL $\neq$ RHL $\therefore f$ is discontinous at $x = 3$.

$\therefore f$ is continous on $R - \{3\}$.

OR

Since f is cts. at $x = 0$,

$\lim_{x \to 0} f(x) = f(0) \Rightarrow \lim_{x \to 0} \frac{1 - \cos 2x}{2x^2} = k$

$\Rightarrow \lim_{x \to 0} \frac{\cancel{2} \sin^2 x}{\cancel{2} x^2} = k \Rightarrow \underline{\underline{k = 1}}$ Ans.

Ans 3

$$f(x) = \begin{cases} -(x-1)-(x+2) & x \le -2 \\ -(x-1)+(x+2) & -2 < x < 1 \\ (x-1)+(x+2) & x \ge 1 \end{cases}$$

$$= \begin{cases} -2x-1 & , x \le -2 \\ 3 & , -2 < x < 1 \\ 2x+1 & , x \ge 1 \end{cases}$$

clearly, f is differentiable everywhere exept possible points of non-differentiability at $x=-2$ and $x=1$.

At $x=-2$

$$LHD = \lim_{h\to 0} \frac{f(-2-h)-f(-2)}{-h} = \lim_{h\to 0} \frac{[-2(-2-h)-1]-3}{-h} = \lim_{h\to 0} \frac{4+2h-1-3}{-h}$$
$$= -2$$

$$RHD = \lim_{h\to 0} \frac{f(-2+h)-f(-2)}{h} = \lim_{h\to 0} \frac{3-3}{h} = 0$$

$\therefore$ f is not diff. at $x=-2$

At $x=1$

$$LHD = \lim_{h\to 0} \frac{f(1-h)-f(1)}{-h} = \lim_{h\to 0} \frac{3-3}{-h} = 0$$

$$RHD = \lim_{h\to 0} \frac{f(1+h)-f(1)}{h} = \lim_{h\to 0} \frac{[2(1+h)+1]-3}{h} = \lim_{h\to 0} \frac{3+2h-3}{h} = 2$$

Since, $LHD \ne RHD \Rightarrow f$ is not diff. at $x=1$

$\therefore$ f is not diff. at $x=1$ and -2

Also, f being sum of two modulus functions, which are both continous, is continous.

Ans 4 (a) $2\sin y.\cos y.\frac{dy}{dx} + (-\sin xy)\left\{x\frac{dy}{dx}+y\right\} = 0$

$\Rightarrow \sin 2y\frac{dy}{dx} - x\sin xy.\frac{dy}{dx} = y\sin xy$

$\Rightarrow \frac{dy}{dx} = \frac{y\sin xy}{\sin 2y - x\sin xy}$ Ans

(b) Let $\frac{5}{13} = \sin\theta$, then $\frac{12}{13} = \cos\theta$, $x = \cos\phi$

$\therefore$ $y = \sin^{-1}(\sin\theta\cos\phi + \cos\theta.\sin\phi)$

$= \sin^{-1}(\sin(\theta+\phi)) = \theta+\phi$

$= \sin^{-1}\frac{5}{13} + \cos^{-1}x$

$\therefore \frac{dy}{dx} = \frac{-1}{\sqrt{1-x^2}}$ Ans