

Answer Key XII A test

Ans 1 (a) $4^{\cot x} \times (-\operatorname{cosec}^2 x) \times \log_e 4$

(b) $y = \cot^{-1} \left(\cot \left(\frac{\pi}{2} - x \right) \right) = \frac{\pi}{2} - x \quad \therefore \frac{dy}{dx} = -1$

(c) $y' = \frac{1}{x + \sqrt{1+x^2}} \left(1 + \frac{2x}{2\sqrt{1+x^2}} \right) = \frac{1}{x + \sqrt{1+x^2}} \left(\frac{\sqrt{1+x^2} + x}{\sqrt{1+x^2}} \right) = \frac{1}{\sqrt{1+x^2}}$

(d) $y = \tan^{-1} \left(\frac{1 + \tan x}{1 - \tan x} \right) = \tan^{-1} \left(\tan \left(\frac{\pi}{4} + x \right) \right) = \frac{\pi}{4} + x \Rightarrow \frac{dy}{dx} = 1$

(e) $\frac{dy}{dx} = \sec^2(\cos^{-1} e^{-x}) \cdot \frac{-1}{\sqrt{1-(e^{-x})^2}} \times e^{-x} \times -1$
 $= \frac{e^{-x}}{\sqrt{1-e^{-2x}}} \sec^2(\cos^{-1} e^{-x})$

Ans 2 when $x \leq -3$, being a sum of modulus and constant functions which are always continuous, f is continuous.

Similar cases for $-3 < x < 3$ and $x \geq 3$ can be discussed.

Possible points of discontinuity are -3 and 3 .

At $x = -3$

$$\text{LHL} = \lim_{h \rightarrow 0} f(-3-h) = \lim_{h \rightarrow 0} |-3-h| + 3 = 6$$

$$\text{RHL} = \lim_{h \rightarrow 0} f(-3+h) = \lim_{h \rightarrow 0} -2(-3+h) = 6$$

Also, $f(-3) = |-3| + 3 = 6 \quad \therefore$ continuous at $x = -3$.

At $x = 3$

$$\text{LHL} = \lim_{h \rightarrow 0} f(3-h) = -2(3-h) = -6$$

$$\text{RHL} = \lim_{h \rightarrow 0} f(3+h) = 6(3+h) + 2 = 20, \text{ Also, } f(3) = 20$$

Since, $\text{LHL} \neq \text{RHL} \quad \therefore f$ is discontinuous at $x = 3$.

$\therefore f$ is continuous on $\mathbb{R} - \{3\}$.

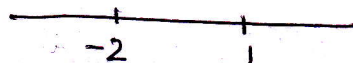
OR

Since f is cts. at $x = 0$,

$$\lim_{x \rightarrow 0} f(x) = f(0) \Rightarrow \lim_{x \rightarrow 0} \frac{1 - \cos 2x}{2x^2} = k$$

$$\Rightarrow \lim_{x \rightarrow 0} \frac{2 \sin^2 x}{2x^2} = k \Rightarrow \underline{k=1} \text{ Ans.}$$

Ans 3 $f(x) = \begin{cases} (x-1) + (x+2) & x \leq -2 \\ -(x-1) + (x+2) & -2 < x < 1 \\ (x-1) + (x+2) & x \geq 1 \end{cases}$



$$= \begin{cases} -2x - 1 & , x \leq -2 \\ 3 & , -2 < x < 1 \\ 2x + 1 & , x \geq 1 \end{cases}$$

clearly, f is differentiable everywhere except possible points of non-differentiability at $x = -2$ and $x = 1$.

At $x = -2$

$$\text{LHD} = \lim_{h \rightarrow 0} \frac{f(-2-h) - f(-2)}{-h} = \lim_{h \rightarrow 0} \frac{[-2(-2-h) - 1] - 3}{-h} = \lim_{h \rightarrow 0} \frac{4+2h-1-3}{-h}$$

$$\text{RHD} = \lim_{h \rightarrow 0} \frac{f(-2+h) - f(-2)}{h} = \lim_{h \rightarrow 0} \frac{3-3}{h} = 0 \quad \text{---} \quad = -2$$

$\therefore f$ is not diff. at $x = -2$

At $x = 1$

$$\text{LHD} = \lim_{h \rightarrow 0} \frac{f(1-h) - f(1)}{-h} = \lim_{h \rightarrow 0} \frac{3-3}{-h} = 0$$

$$\text{RHD} = \lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h} = \lim_{h \rightarrow 0} \frac{[2(1+h) + 1] - 3}{h} = \lim_{h \rightarrow 0} \frac{3+2h-3}{h} = 2$$

Since, $\text{LHD} \neq \text{RHD} \Rightarrow f$ is not diff. at $x = 1$

$\therefore f$ is not diff. at $x = 1$ and -2

Also, f being sum of two modulus functions, which are both continuous, is continuous.

Ans 4 (a) $2 \sin y \cdot \cos y \cdot \frac{dy}{dx} + (-\sin xy) \left\{ x \frac{dy}{dx} + y \right\} = 0$

$$\Rightarrow \sin 2y \frac{dy}{dx} - x \sin xy \cdot \frac{dy}{dx} = y \sin xy$$

$$= \frac{dy}{dx} = \frac{y \sin xy}{\sin 2y - x \sin xy} \quad \text{Ans}$$

(b) let $\frac{5}{13} = \sin \theta$, then $\frac{12}{13} = \cos \theta$, $x = \sin \phi$

$$\therefore y = \sin^{-1}(\sin \theta \cos \phi + \cos \theta \cdot \sin \phi)$$

$$= \sin^{-1}(\sin(\theta + \phi)) = \theta + \phi$$

$$= \sin^{-1} \frac{5}{13} + \cos^{-1} x$$

$$\therefore \frac{dy}{dx} = \frac{-1}{\sqrt{1-x^2}} \text{ Ans}$$